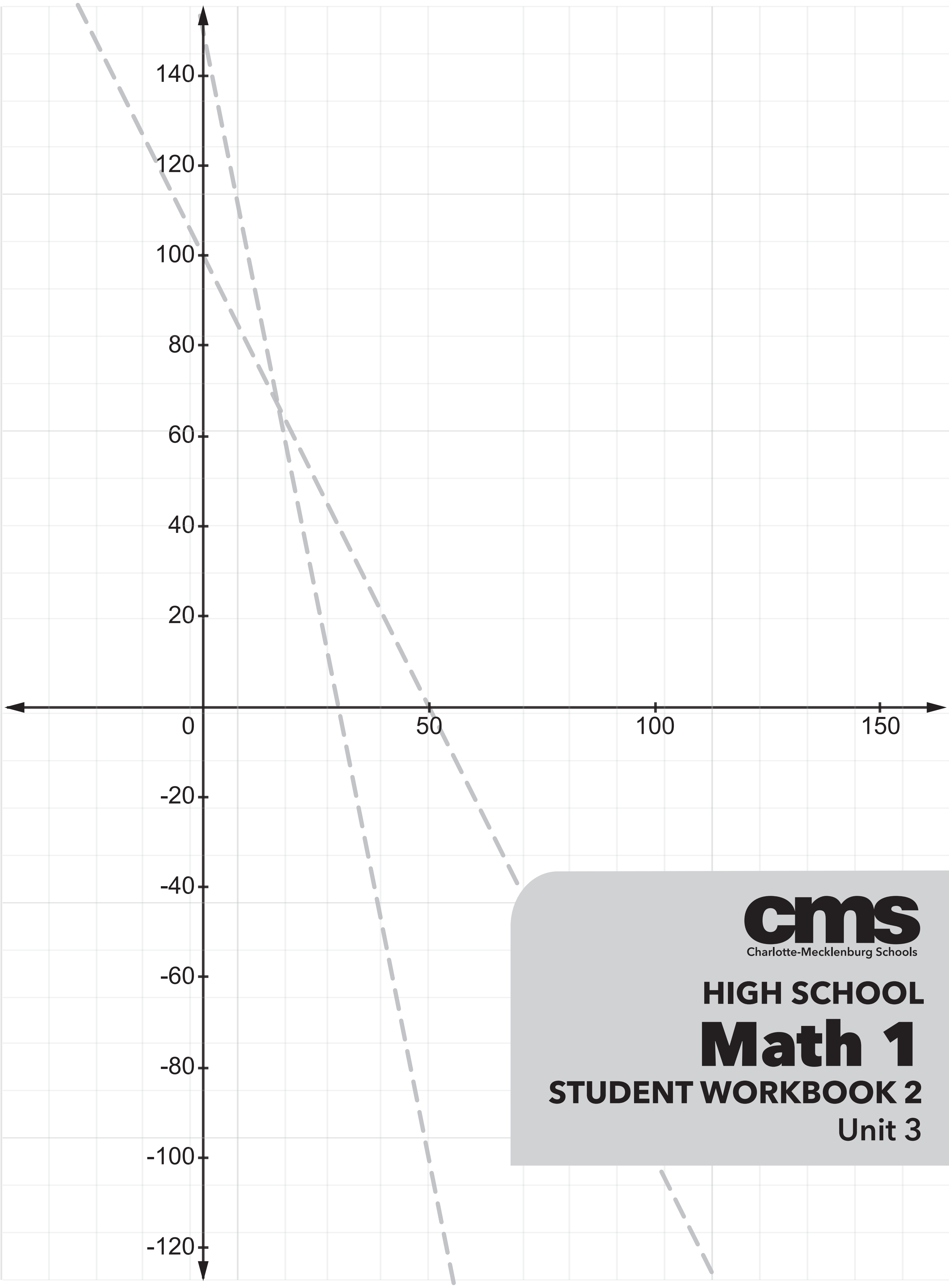


cms
Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

STUDENT WORKBOOK 2

Unit 3

Table of Contents

Lesson 1: Equations and Their Graphs

Learning Targets

- When given the graph of a linear equation, I can explain the meaning of the points on the graph in terms of the situation it represents.
- I understand how the coordinates of the points on the graph of a linear equation are related to the equation.
- I can use graphing technology to graph linear equations and identify solutions to the equations.

Bridge

Solve each equation mentally.

a. $100 = 10(x - 5)$

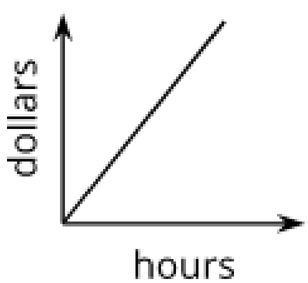
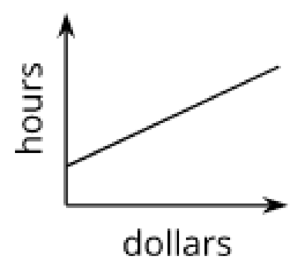
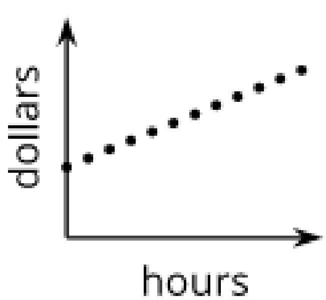
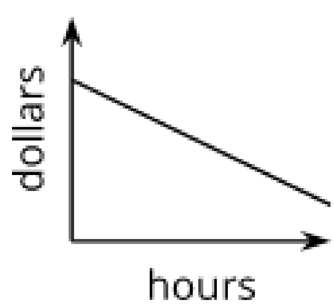
b. $300 = 30(x - 5)$

c. $15 - 971 = x - 4 - 971$

d. $\frac{10}{7} = \frac{1}{7}(x - 19)$

Warm-up: Hours and Dollars

Which one doesn't belong? Explain your reasoning.

a. 	b. 
c. 	d. 

Activity 1: A Family Reunion

To get food for a family reunion, Clare went to the local grocery store where she can buy any quantity of a product and the prices are usually good.

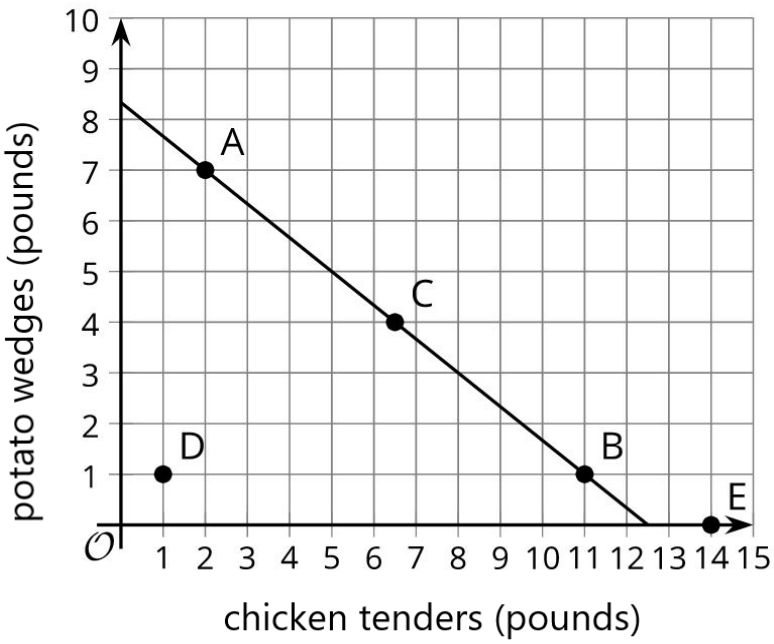
Clare purchased chicken tenders at $6 per pound and potato wedges at $9 per pound. She spent $75 before tax.

1. If she bought 2 pounds of chicken tenders, how many pounds of potato wedges did she buy?

2. If she bought 1 pound of potato wedges, how many pounds of chicken tenders did she buy?

3. Write an equation that describes the relationship between pounds of potato wedges and pounds of chicken tenders that Clare bought and the dollar amount that she paid. Be sure to specify what the variables represent.

4. Here is a graph that represents the quantities in this situation.



a. Choose any labeled point on the line, state its coordinates, and explain what it tells us.

b. Choose any labeled point that is *not* on the line, state its coordinates, and explain what it tells us.

Activity 2: Graph It!

A student has a savings account with $475 in it. She deposits $125 of her paycheck into the account every week.

1. How much will be in the account after 3 weeks?

2. How long will it take before she has $1,350?

3. Write an equation that represents the relationship between the dollar amount in her account and the number of weeks of saving.

4. Graph your equation using graphing technology. Note the points on the graph that represent the amount after 3 weeks and the week she has $1,350. Write down the coordinates.

5. She determines her goal is to save $7,000 for college. How long will it take her to reach her goal?

Are You Ready For More?

1. A 450-gallon tank full of water is draining at a rate of 20 gallons per minute.
    a. How many gallons will be in the tank after 7 minutes?
    b. How long will it take for the tank to have 200 gallons?
    c. Write an equation that represents the relationship between the gallons of water in the tank and minutes the tank has been draining.
    d. Graph your equation using graphing technology. Mark the points on the graph that represent the gallons after 7 minutes and the time when the tank has 200 gallons. Write down the coordinates.
    e. How long will it take until the tank is empty?
2. Write an equation that represents the relationship between the gallons of water in the tank and *hours* the tank has been draining.

3. Write an equation that represents the relationship between the gallons of water in the tank and *seconds* the tank has been draining.

4. Graph each of your new equations. In what way are all of the graphs the same? In what way are they all different?

5. How would these graphs change if we used quarts of water instead of gallons? What would stay the same?

Lesson Debrief

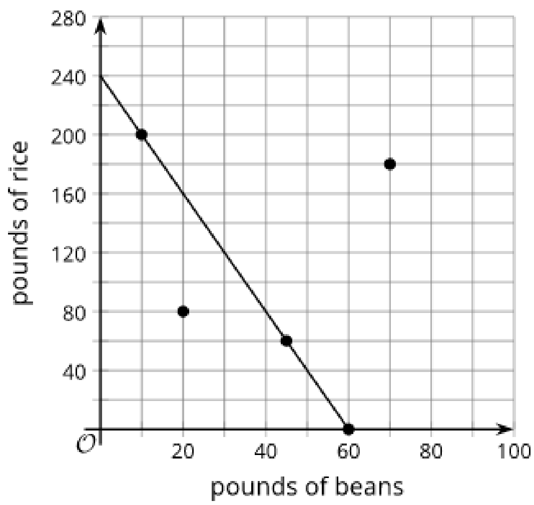
Lesson 1 Summary and Glossary

Like an equation, a graph can give us information about the relationship between quantities and the constraints on them.

Suppose we are buying beans and rice to feed a large gathering of people, and we plan to spend $120 on the two ingredients. Beans cost $2 per pound and rice costs $0.50 per pound.

If x represents pounds of beans and y represents pounds of rice, the equation $2x + 0.50y = 120$ can represent the constraints in this situation.

The graph of $2x + 0.50y = 120$ shows a straight line.



Each point on the line is a pair of x- and y-values that makes the equation true and is thus a solution. It is also a pair of values that satisfies the constraints in the situation.

- The point $(10, 200)$ is on the line. If we buy 10 pounds of beans and 200 pounds of rice, the cost will be $2(10) + 0.50(200)$, which equals 120.
- The points $(60, 0)$ and $(45, 60)$ are also on the line. If we buy only beans—60 pounds of them—and no rice, we will spend $120. If we buy 45 pounds of beans and 60 pounds of rice, we will also spend $120.

What about points that are *not* on the line? They are not solutions because they don't satisfy the constraints, but they still have meaning in the situation.

- The point $(20, 80)$ is not on the line. Buying 20 pounds of beans and 80 pounds of rice costs $2(20) + 0.50(80)$ or $80, which does not equal $120. This combination costs less than what we intend to spend.
- The point $(70, 180)$ means that we buy 70 pounds of beans and 180 pounds of rice. It will cost $2(70) + 0.50(180)$ or $230, which is over our budget of $120.

Unit 3 Lesson 1 Practice Problems

1. Select **all** the points that are on the graph of the equation $4y - 6x = 12$.

   a. $(-4, -3)$

   b. $(-1, 1.5)$

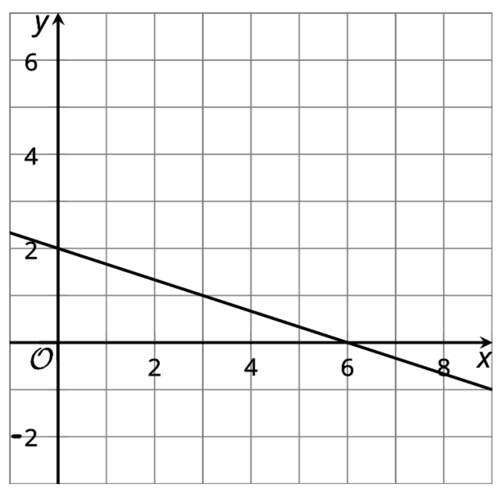
   c. $(0, -2)$

   d. $(0, 3)$

   e. $(3, -4)$

   f. $(6, 4)$

2. Here is a graph of the equation $x + 3y = 6$. Select **all** coordinate pairs that represent a solution to the equation.



   a. $(0, 2)$

   b. $(0, 6)$

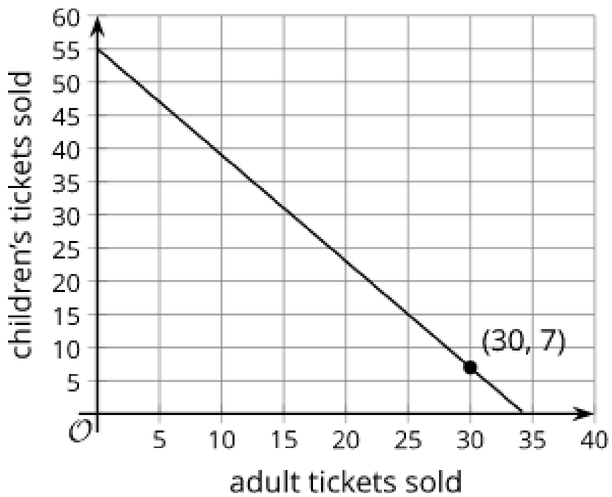
   c. $(2, 6)$

   d. $(3, 1)$

   e. $(4, 1)$

   f. $(6, 2)$

3. A theater is selling tickets to a play. Adult tickets cost \$8 each, and children's tickets cost \$5 each. They collect \$275 after selling x adult tickets and y children's tickets.



What does the point $(30, 7)$ mean in this situation?

4. (*Technology required.*) Priya starts with \$50 in her bank account. She then deposits \$20 each week for 12 weeks.

   a. Write an equation that represents the relationship between the dollar amount in her bank account and the number of weeks of saving.

   b. Graph your equation using Desmos or other graphing technology. Mark the point on the graph that represents the amount after 3 weeks.

   c. How many weeks does it take her to have \$250 in her bank account? Mark this point on the graph.

5. A student on the cross-country team runs 30 minutes a day as a part of her training.

   Write an equation to describe the relationship between the distance she runs in miles, D, and her running speed, in miles per hour, when she runs:

   a. at a constant speed of 4 miles per hour for the entire 30 minutes

   b. at a constant speed of 5 miles per hour the first 20 minutes, and then at 4 miles per hour the last 10 minutes

   c. at a constant speed of 6 miles per hour the first 15 minutes, and then at 5.5 miles per hour for the remaining 15 minutes

   d. at a constant speed of a miles per hour the first 6 minutes, and then at 6.5 miles per hour for the remaining 24 minutes

   e. at a constant speed of 5.4 miles per hour for m minutes, and then at b miles per hour for n minutes

   (From Unit 2)

6. In the 21st century, people measure length in feet and meters. At various points in history, people measured length in hands, cubits, and paces. There are 9 hands in 2 cubits. There are 5 cubits in 3 paces.

   a. Write an equation to express the relationship between hands, h, and cubits, c.

   b. Write an equation to express the relationship between hands, h, and paces, p.

   (From Unit 2)

7. The table shows the amount of money, A, in a savings account after m months.

   Select **all** the equations that represent the relationship between the amount of money, A, and the number of months, m.

   a. $A = 100m$
   b. $A = 100(m-5)$
   c. $A-700 = 100m$
   d. $A-1,200 = 100m$
   e. $A = 700 + 100m$
   f. $A = 1200 + 100m$
   g. $A = 1,200 + 100(m-5)$

Number of months	Dollar amount
5	1,200
6	1,300
7	1,400
8	1,500

(From Unit 2)

8. Solve each equation for y.

   a. $(y-10) = -3(x-2)$
   b. $(y-2) = 3(x+1)$
   c. $(y-2) = \frac{1}{3}(x-3)$

(From Unit 2)

9. During the month of August, the mean of the daily rainfall in one city was 0.04 inches with a standard deviation of 0.15 inches. In another city, the mean of the daily rainfall was 0.01 inches with a standard deviation of 0.05 inches.

   What does the given information tell you about the two cities' patterns of precipitation in the month of August? Explain your reasoning.

(From Unit 1)

10. Solve each equation.[1]

a. $2(x-3)=14$

b. $-5(x-1)=40$

c. $12(x+10)=24$

d. $\frac{1}{6}(x+6)=11$

e. $\frac{5}{7}(x-9)=25$

(Addressing NC.7.EE.4)

Lesson 2: Connecting Equations to Graphs (Part One)

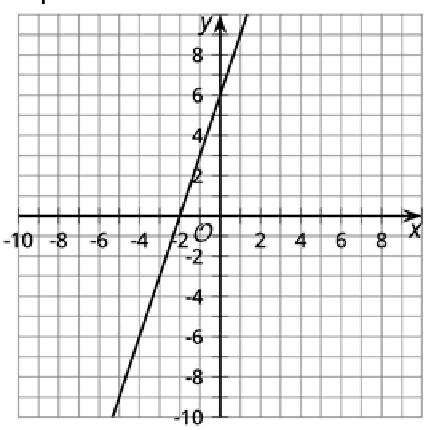
Learning Targets

- I can describe the connections between an equation of the form $ax + by = c$, the features of its graph, and the rate of change in the situation.
- I can graph a linear equation of the form $ax + by = c$.
- I understand that rewriting the equation for a line in different forms can make it easier to find certain kinds of information about the relationship and about the graph.

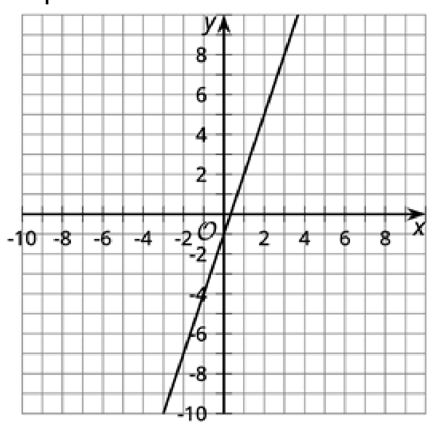
Bridge

Here are the graphs of four equations:

Graph A



Graph B

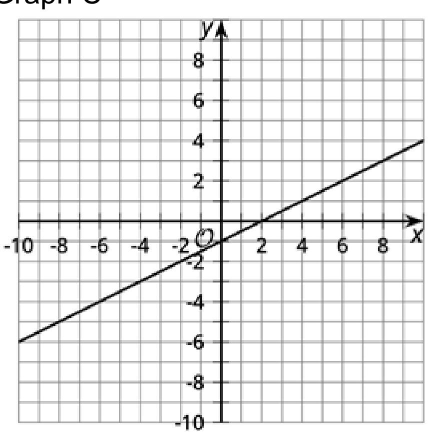


1. Which graphs have a slope of 3?

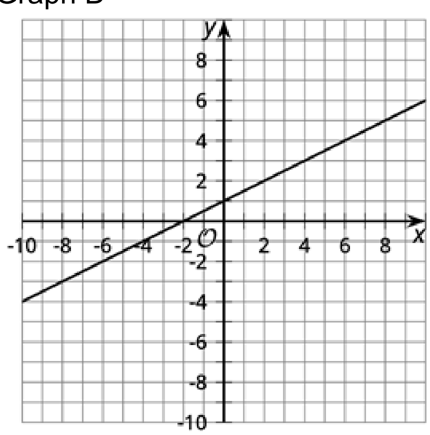
2. Which graphs have a slope of ½?

3. Which graphs have a y-intercept of -1?

Graph C



Graph D



4. Which graphs have an x-intercept of -2?

Warm-up: Game and Rides

Jada has $20 to spend on games and rides at a carnival. Games cost $1 each and rides are $2 each.

1. Given this situation, what can Jada do with her $20?

2. Which equation represents the relationship between the number of games, x, and the number of rides, y, that Jada could go on if she spends all her money?

   a. $x + y = 20$

   b. $2x + y = 20$

   c. $x + 2y = 20$

3. Explain what each of the other two equations could mean in this situation.

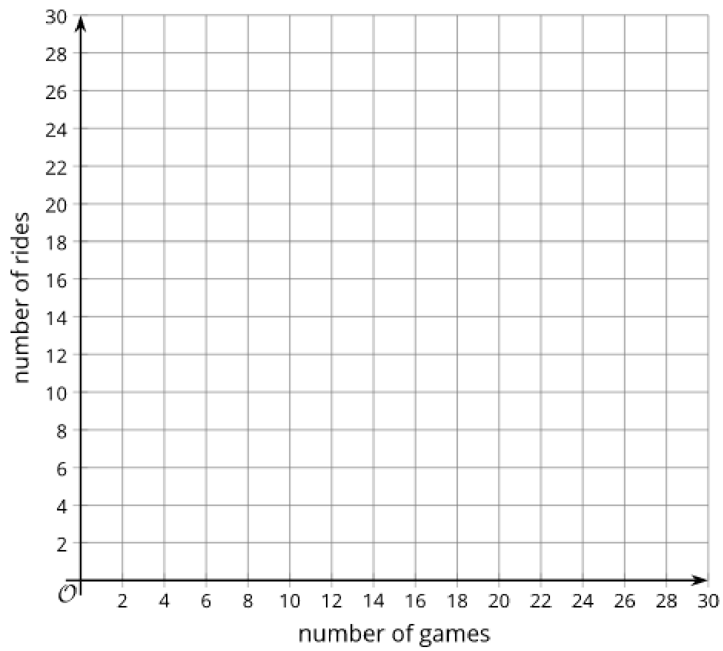
Activity 1: Graphing Games and Rides

Here are the three equations. Each represents the relationship between the number of games, x, the number of rides, y, and the dollar amount a student is spending on games and rides at a different carnival.

Equation 1: $x + y = 20$ Equation 2: $2.50x + y = 15$ Equation 3: $x + 4y = 28$

As a group, choose one of the equations above and answer the questions below based on that equation.

Equation ____________________:



1. What's the number of rides the student could get on if they don't play any games? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.

2. What's the number of games the student could play if they don't get on any rides? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.

3. Draw a line to connect the two points you've drawn.

4. Complete the sentences:

   "If the student played no games, they can get on _____ rides. For every additional game that the student plays, x, the possible number of rides, y, ___________ (increases or decreases) by ______________."

5. What is the slope of your graph? Where does the graph intersect the vertical axis?

6. Rearrange the equation to solve for y.

7. What connections, if any, do you notice between your new equation and the graph?

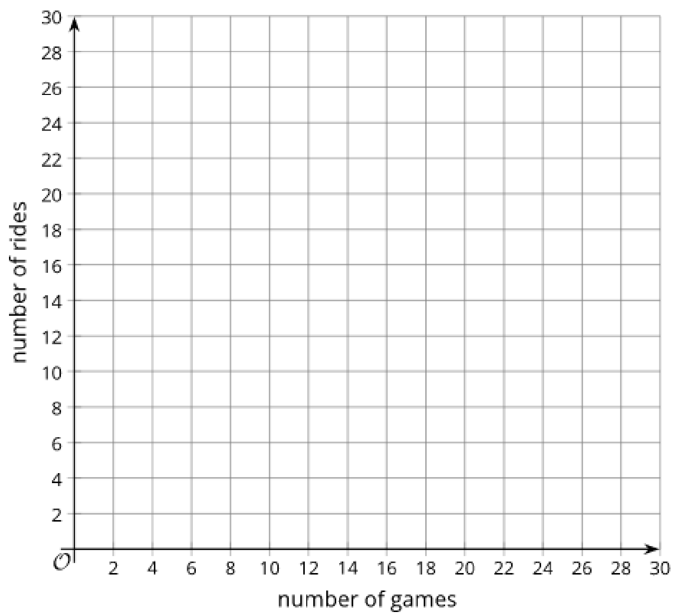
Are You Ready For More?

Here are the same three equations as the task statement above. Each represents the relationship between the number of games, x, the number of rides, y, and the dollar amount a student is spending on games and rides at a different carnival.

Equation 1: $x + y = 20$ Equation 2: $2.50x + y = 15$ Equation 3: $x + 4y = 28$

Choose a different equation above and answer the questions below based on that equation.

Equation ________________________:



1. What's the number of rides the student could get on if they don't play any games? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.
2. What's the number of games the student could play if they don't get on any rides? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.
3. Draw a line to connect the two points you've drawn.
4. Complete the sentences:

   "If the student played no games, they can get on ____ rides. For every additional game that the student plays, x, the possible number of rides, y, __________ (increases or decreases) by ______________."
5. What is the slope of your graph? Where does the graph intersect the vertical axis?
6. Rearrange the equation to solve for y.
7. What connections, if any, do you notice between your new equation and the graph?

Lesson Debrief

Lesson 2 Summary and Glossary

Linear equations can be written in different forms. Some forms allow us to better see the relationship between quantities or to find features of the graph of the equation.

Suppose an athlete wishes to burn 700 calories a day by running and swimming. He burns 17.5 calories per minute of running and 12.5 calories per minute of freestyle swimming.

Let x represent the number of minutes of running and y the number of minutes of swimming. To represent the combination of running and swimming that would allow him to burn 700 calories, we can write:

$$17.5x + 12.5y = 700$$

We can reason that the more minutes he runs, the fewer minutes he has to swim to meet his goal. In other words, as x increases, y decreases. If we graph the equation, the line will slant down from left to right.

If the athlete only runs and doesn't swim, how many minutes would he need to run?

Let's substitute 0 for y to find x:

$$\begin{aligned} 17.5x + 12.5(0) &= 700 \\ 17.5x &= 700 \\ x &= \tfrac{700}{17.5} \\ x &= 40 \end{aligned}$$

On a graph, this combination of times is the point $(40, 0)$, which is the x-intercept.

If he only swims and doesn't run, how many minutes would he need to swim?

Let's substitute 0 for x to find y:

$$17.5(0) + 12.5y = 700$$
$$12.5y = 700$$
$$y = \frac{700}{12.5}$$
$$y = 56$$

On a graph, this combination of times is the point $(0, 56)$, which is the y-intercept.

If the athlete wants to know how many minutes he would need to swim if he runs for 15 minutes, 20 minutes, or 30 minutes, he can substitute each of these values for x in the equation and find y. Or, he can first solve the equation for y:

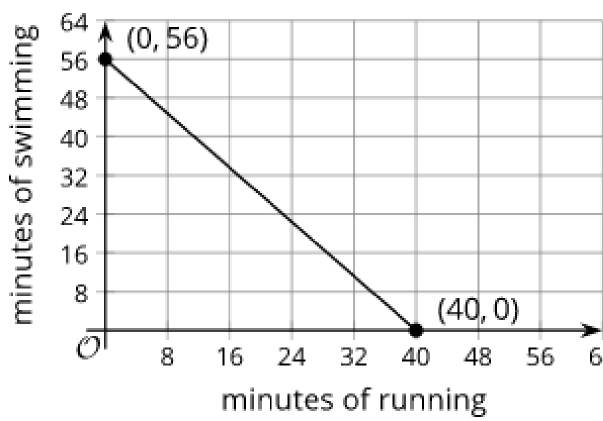
$$27.5x + 12.5y = 700$$
$$12.5y = 700 - 17.5x$$
$$y = \frac{700 - 17.5x}{12.5}$$
$$y = 56 - 1.4x$$

Notice that $y = 56 - 1.4x$, or $y = -1.4x + 56$, is written in slope-intercept form.

- The coefficient of x, -1.4, is the slope of the graph. It means that as x increases by 1, y falls by 1.4. For every additional minute of running, the athlete can swim 1.4 fewer minutes.
- The constant term, 56, tells us where the graph intersects the y-axis. It tells us the number of minutes the athlete would need to swim if he does no running.

The first equation we wrote, $17.5x + 12.5y = 700$, is a linear equation in standard form. In general, it is expressed as $Ax + By = C$, where x and y are variables, and A, B, and C are numbers.

The two equations, $17.5x + 12.5y = 700$ and $y = -1.4x + 56$, are equivalent, so they have the same solutions and the same graph.



Unit 3 Lesson 2 Practice Problems

1. A little league baseball team is ordering hats. The graph shows the relationship between the total cost, in dollars, and the number of hats ordered.

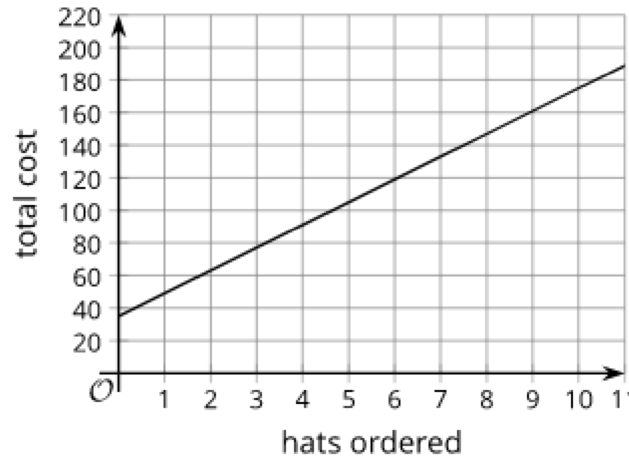
   What does the **slope** of the graph tell us in this situation?

   a. It tells us that there is a fixed cost of approximately $35 for ordering hats.

   b. It tells us the amount that the total cost increases for each additional hat ordered.

   c. It tells us that when 9 hats are ordered, the total cost is approximately $160.

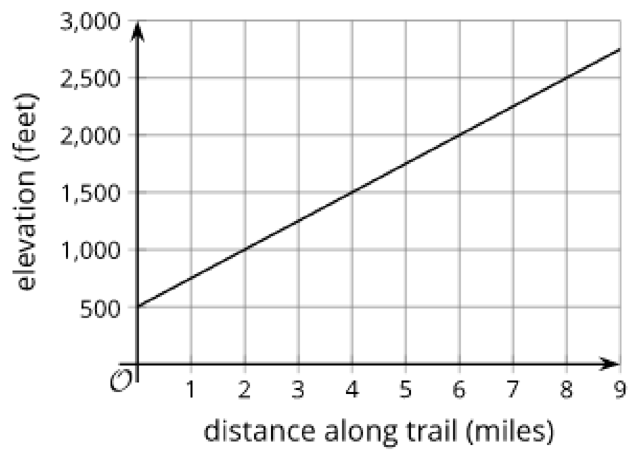
   d. It tells us that when the number of hats ordered increases by 10, the total cost increases by approximately $175.



2. A group of hikers is progressing steadily along an uphill trail. The graph shows their elevation (or height above sea level), in feet, at each distance from the start of the trail, in miles.

   a. What is the slope of the graph? Show your reasoning.

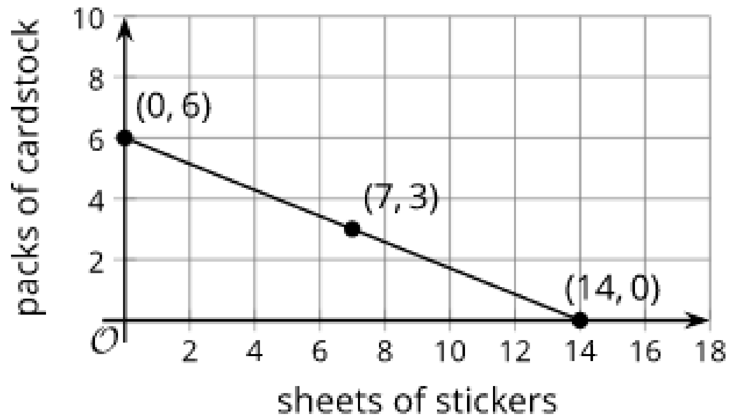
   b. What does the slope tell us about this situation?



   c. Write an equation that represents the relationship between the hikers' distance from the start of the trail, x, and their elevation, y.

   d. Does the equation $y - 250x = 500$ represent the same relationship between the distance from the start of the trail and the elevation? Explain your reasoning.

3. A kindergarten teacher bought $21 worth of stickers and cardstock for his class. The stickers cost $1.50 a sheet, and the cardstock cost $3.50 per pack. The equation $1.5s + 3.5c = 21$ represents the relationship between sheets of stickers, s, packs of cardstock, c, and the dollar amount the kindergarten teacher spent on these supplies.



   a. Explain how we can tell that this graph represents the given equation.

   b. What do the vertical and horizontal intercepts, $(0, 6)$ and $(14, 0)$, mean in this situation?

4. Andre bought a new bag of cat food. The next day, he opened it to feed his cat. The graph shows how many ounces were left in the bag on the days after it was bought.

   a. How many ounces of food were in the bag 12 days after Andre bought it?

   b. How many days did it take for the bag to contain 16 ounces of food?

   c. How much did the bag weigh before it was opened?

   d. About how many days did it take for the bag to be empty?

(From Unit 3, Lesson 1)

5. In physics, the equation $PV = nRT$ is called the ideal gas law. It is used to approximate the behavior of many gases under different conditions.

   P, V, and T represent pressure, volume, and temperature, n represents the number of moles of gas, and R is a constant for the ideal gas.

   Which equation is solved for T?

   a. $\frac{PV}{R} = nT$
   b. $\frac{PV}{nR} = T$
   c. $T = PV - nR$
   d. $PVnR = T$

   (From Unit 2)

6. To raise funds for uniforms and travel expenses, the soccer team is holding a car wash in a part of town with a lot of car and truck traffic. The team spent \$90 on supplies like sponges and soap. They plan to charge \$10 per car and \$20 per truck. Their goal is to raise \$460.

   How many cars do they have to wash if they washed the following numbers of trucks?

   a. 4 trucks
   b. 15 trucks
   c. 21 trucks
   d. 27 trucks
   e. t trucks

   (From Unit 2)

7. During the Middle Ages, people often used grains, scruples, and drahms to measure the weights of different medicines.

   If 120 grains are equivalent to 6 scruples and 6 scruples are equivalent to 2 drahms, how many drahms are equivalent to 300 grains? Explain your reasoning. If you get stuck, try creating a table.

   (From Unit 2)

8. Explain why the equation $2(3x-5)=6x+8$ has no solutions.

   (From Unit 2)

9. Consider the equation $3a+0.1n=123$. If we solve this equation for n, which equation would result?

   a. $0.1n=123-3a$

   b. $n=123-3a-0.1$

   c. $n=1,230-30a$

   d. $\frac{3a-123}{0.1}=n$

   (From Unit 2)

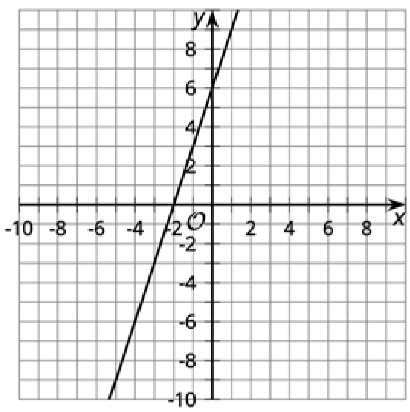
10. Diego is buying shrimp and rice to make dinner. Shrimp costs $6.20 per pound and rice costs $1.25 per pound. Diego spent $10.55 buying shrimp and rice. The relationship between pounds of shrimp s, pounds of rice r, and the total cost is represented by the equation $6.20s + 1.25r = 10.55$.

    Write an equation that makes it easy to find the number of pounds of rice if we know the number of pounds of shrimp purchased.

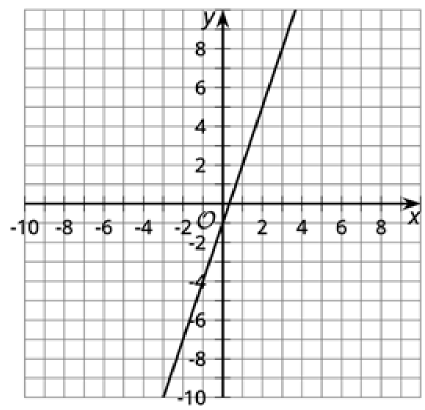
    (From Unit 2)

Use the following graphs to answer questions 11 and 12.

Graph A



Graph B



11. Graph A represents the equation $2y - 6x = 12$. Which other equations could graph A represent?

    a. $y - 3x = 6$

    b. $y = 3x + 6$

    c. $y = -3x + 6$

    d. $2y = -6x + 12$

    e. $4y - 12x = 12$

    f. $4y - 12x = 24$

    (Addressing NC.8.F.4)

12. Write three equations that graph B could represent.

    a. ______________________ b. ______________________ c. ______________________

    (Addressing NC.8.F.4)

Lesson 3: Connecting Equations to Graphs (Part Two)

Learning Targets

- I can use a variety of strategies to find the slope and vertical intercept of the graph of a linear equation given in different forms.
- I can take an equation of the form $ax + by = c$ and rearrange it into the equivalent form $y = mx + b$.

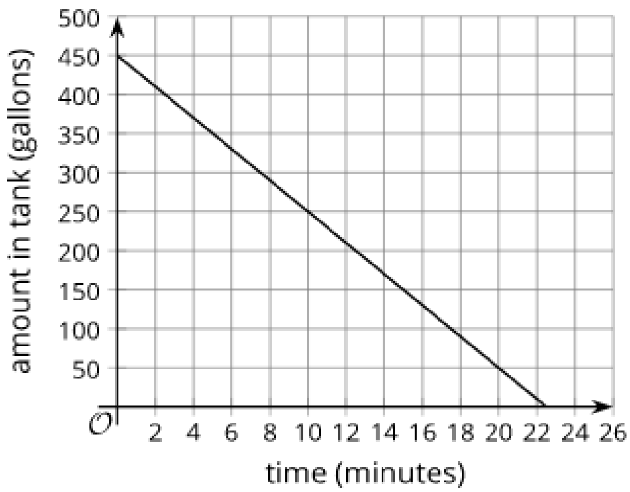
Warm-up: Rewrite These!

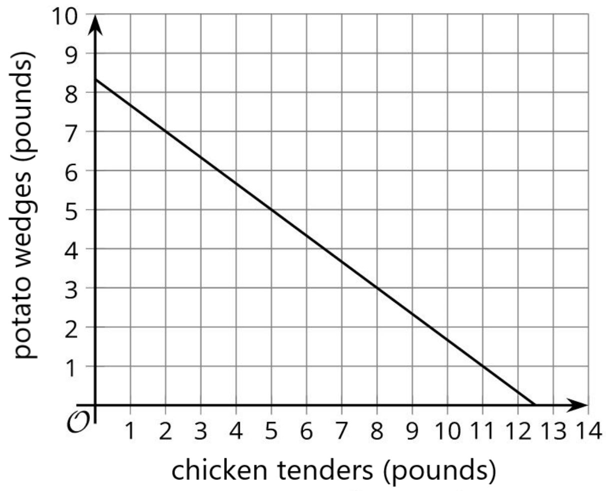
Rewrite each quotient as a sum or a difference.

1. $\frac{4x-10}{2}$
2. $\frac{5(x+10)}{25}$
3. $\frac{1-50x}{-2}$
4. $\frac{-\frac{1}{5}x+5}{2}$

Activity 1: Graphs of Two Equations

Here are two graphs that represent situations you may have seen in earlier activities.





1. The first graph represents $a = 450 - 20t$, which describes the relationship between gallons of water in a tank and time in minutes.
   a. Where on the graph can we see the 450? Where can we see the -20?

   b. What do these numbers mean in this situation?

2. The second graph represents $6x + 9y = 75$. It describes the relationship between pounds of chicken tenders and potato wedges and the dollar amount Clare spent on them.

   Suppose a classmate says, "I am not sure the graph represents $6x + 9y = 75$ because I don't see the 6, 9, or 75 on the graph." How would you show your classmate that the graph indeed represents this equation?

Activity 2: Slope Match

Match each of the equations with the slope m and y-intercept of its graph.

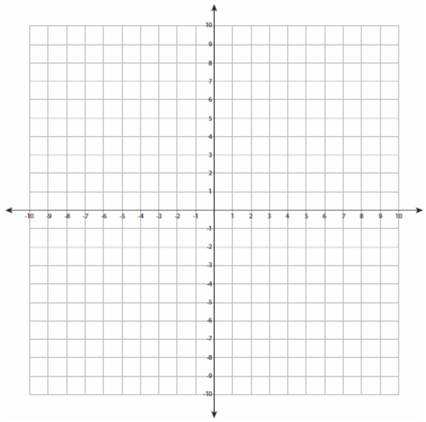
Equation	Slope and y-intercept
1. $-4x+3y=3$	a. $m=3$, $y-int=(0,1)$
2. $12x-4y=8$	b. $m=\frac{4}{3}$, $y-int=(0,1)$
3. $8x+2y=16$	c. $m=\frac{4}{3}$, $y-int=(0,-2)$
4. $-x+\frac{1}{3}y=\frac{1}{3}$	d. $m=-4$, $y-int=(0,8)$
5. $-4x+3y=-6$	e. $m=3$, $y-int=(0,-2)$

Are You Ready For More?

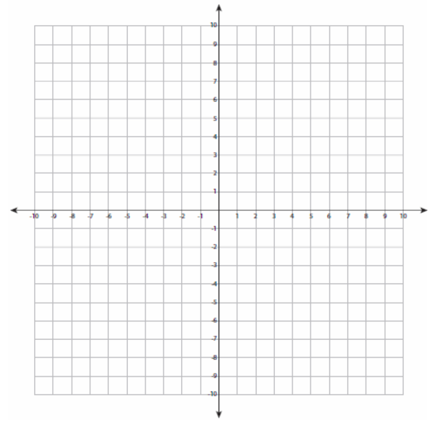
Each equation in Activity 2 is in the form $Ax + By = C$.

1. For each equation in Activity 2, graph the equation and on the same coordinate plane graph the line passing through $(0,0)$ and (A,B). What is true about each pair of lines?

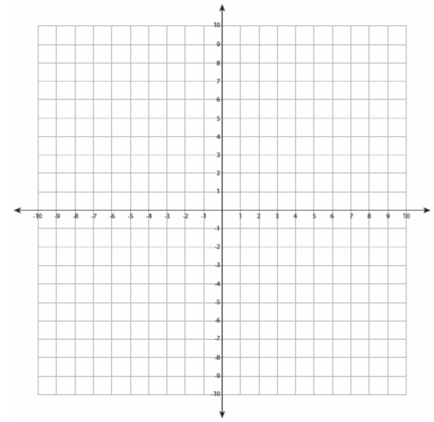
Equation 1



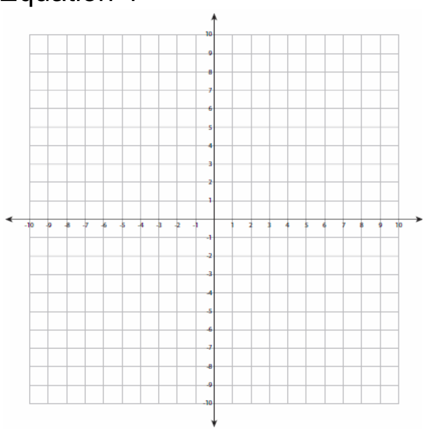
Equation 2



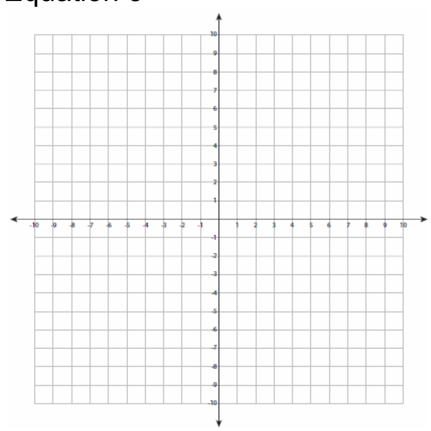
Equation 3



Equation 4



Equation 5



2. What are the coordinates of the x-intercept and y-intercept in terms of A, B, and C?

Lesson Debrief

Lesson 3 Summary and Glossary

Here are two situations and two equations that represent them.

Situation 1: Mai receives a $40 bus pass. Each school day, she spends $2.50 to travel to and from school.

Let d be the number of school days since Mai received a pass and b the balance or dollar amount remaining on the pass.

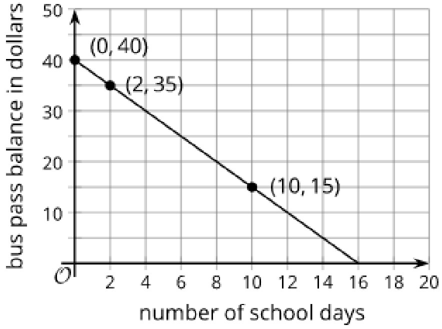
$$b = 40 - 2.50d$$

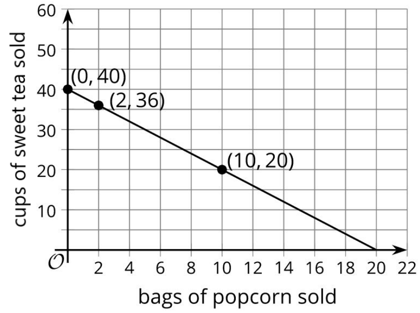
Situation 2: A student club is raising money by selling popcorn and sweet tea. The club is charging $3 per bag of popcorn and $1.50 per cup of sweet tea, and plans to make $60.

Let p be the bags of popcorn sold and t the cups of sweet tea sold.

$$3p + 1.50t = 60$$

Here are graphs of the equations. On each graph, the coordinates of some points are shown.





The 40 in the first equation can be observed on the graph and the -2.50 can be found with a quick calculation. The graph intersects the vertical axis at 40 and the -2.50 is the slope of the line. Every time d increases by 1, b decreases by 2.50. In other words, with each passing school day, the dollar amount in Mai's bus pass drops by 2.50.

The numbers in the second equation are not as apparent on the graph. The values where the line intersects the vertical and horizontal axes, 40 and 20, are not in the equation. We can, however, reason about where they come from.

- If p is 0 (no popcorn is sold), the club would need to sell 40 cups of sweet tea to make \$60 because $40(1.50) = 60$.
- If t is 0 (no sweet tea is sold), the club would need to sell 20 bags of popcorn to make \$60 because $20(3) = 60$.

What about the slope of the second graph? We can compute it from the graph, but it is not shown in the equation $3p + 1.50t = 60$.

Notice that in the first equation, the variable b was isolated. Let's rewrite the second equation and isolate t:

$$3p + 1.50t = 60$$
$$1.50t = 60 - 3p$$
$$t = \frac{60-3p}{1.50}$$
$$t = 40 - 2p$$

Now the numbers in the equation can be more easily related to the graph: The 40 is where the graph intersects the vertical axis and the -2 is the slope. The slope tells us that as p increases by 1, t falls by 2. In other words, for every additional bag of popcorn sold, the club can sell 2 fewer cups of sweet tea.

Unit 3 Lesson 3 Practice Problems

1. What is the slope of the graph of $5x - 2y = 20$?
   a. -10
   b. $\frac{-2}{5}$
   c. $\frac{5}{2}$
   d. 5

2. What is the y-intercept of each equation?
   a. $y = 6x + 2$
   b. $10x + 5y = 30$
   c. $y - 6 = 2(3x - 4)$

3. Han wanted to find the intercepts of the graph of the equation $10x + 4y = 20$. He decided to put the equation in slope-intercept form first. Here is his work:

$$10x + 4y = 20$$
$$4y = 20 - 10x$$
$$y = 5 - 10x$$

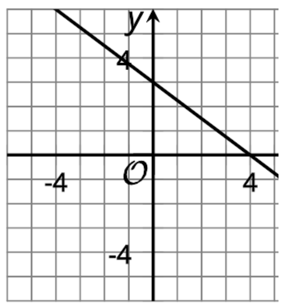
   He concluded that the x-intercept is $(\frac{1}{2}, 0)$ and the y-intercept is $(0, 5)$.

   a. What error did Han make?

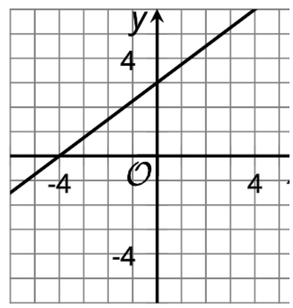
   b. What are the x- and y-intercepts of the line? Explain or show your reasoning.

4. Which graph represents the equation $12 = 3x + 4y$? Explain how you know.

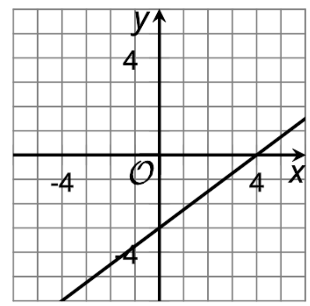
a.



b.

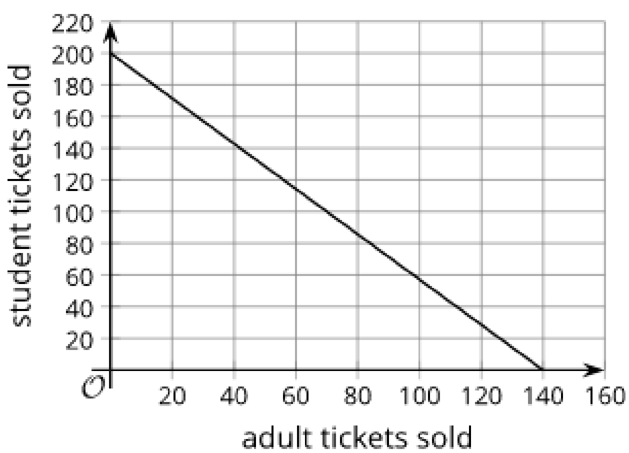


c.



5. A school sells adult tickets and student tickets for a play. It collects $1,400 in total.

The graph shows the possible combinations of the number of adult tickets sold and the number of student tickets sold.



What does the vertical intercept (0, 200) tell us in this situation?

a. It tells us the decrease in the sale of adult tickets for each student ticket sold.

b. It tells us the decrease in the sale of student tickets for each adult ticket sold.

c. It tells us that if no adult tickets were sold, then 200 student tickets were sold.

d. It tells us that if no student tickets were sold, then 200 adult tickets were sold.

(From Unit 3, Lesson 2)

6. Clare knows that Priya has a bunch of nickels and dimes in her pocket and that the total amount is $1.25.

   a. Find one possibility for the number of nickels and number of dimes that could be in Priya's pocket.

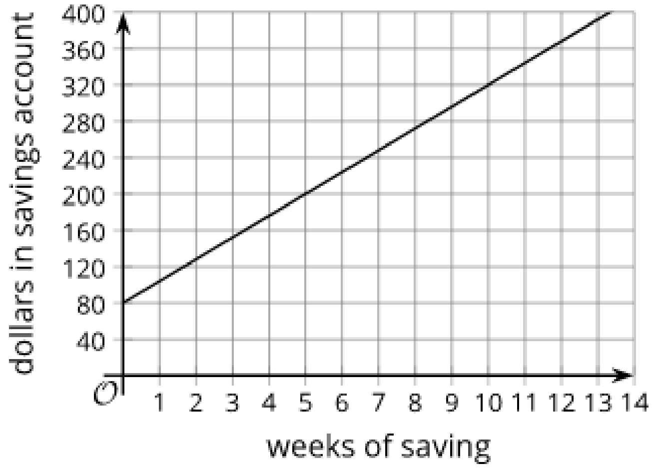
   b. Write an equation that describes the relationship between the number of dimes and the number of nickels in Priya's pocket.

   c. Explain what the point $(13, 6)$ means in this situation.

   d. Is the point $(13, 6)$ a solution to the equation you wrote? Explain your reasoning.

(From Unit 2)

7. The graph shows how much money Priya has in her savings account weeks after she started saving on a regular basis.



a. How much money does Priya have in the account after 10 weeks?

b. How long did it take her to save $200?

c. How much money did Priya have in her savings account when she started to save regularly?

d. Write an equation to represent the dollar amount in her savings account and the number of weeks of saving. Be sure to specify what each variable represents.

(From Unit 2)

8. Noah has a coin jar containing d dimes and q quarters worth a total of \$5.00.

   Select **all** the equations that represent this situation.

   a. $d + q = 5$

   b. $d + q = 500$

   c. $0.1d + 0.25q = 5$

   d. $10d + 25q = 500$

   e. $d = 50$

   f. $q = 20$

   (From Unit 2)

9. Noah orders an extra-large pizza. It costs $\$12.49$ for the pizza plus $\$1.50$ for each topping. He orders an extra-large pizza with t toppings that costs a total of d dollars.

   Select **all** of the equations that represent the relationship between the number of toppings t and total cost d of the pizza with t toppings.

   a. $12.49 + t = d$

   b. $12.49 + 1.50t = d$

   c. $12.49 + 1.50d = t$

   d. $12.49 = d + 1.50t$

   e. $t = \frac{d - 12.49}{1.5}$

   f. $t = d - \frac{12.49}{1.5}$

   (From Unit 2)

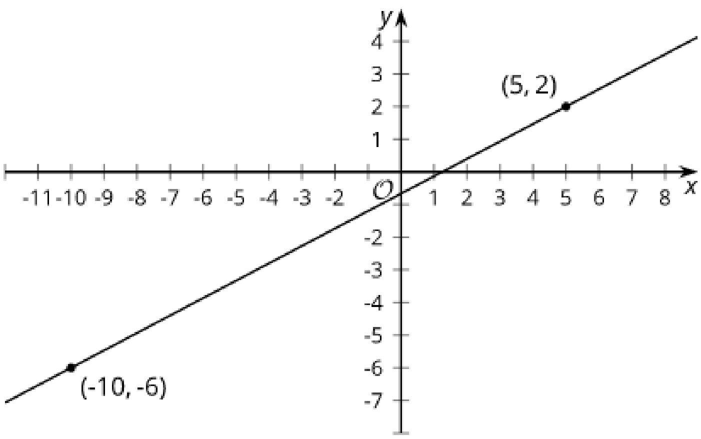
Lesson 4: Equations of Lines

Learning Target

- I can use the definition of slope to write the equation for a line in point-slope form.

Warm-up: Remembering Slope

The slope of the line in the image is $\frac{8}{15}$. Explain how you know this is true.



Activity 1: Building an Equation for a Line

1. Find the slope of the line between the following sets of points:

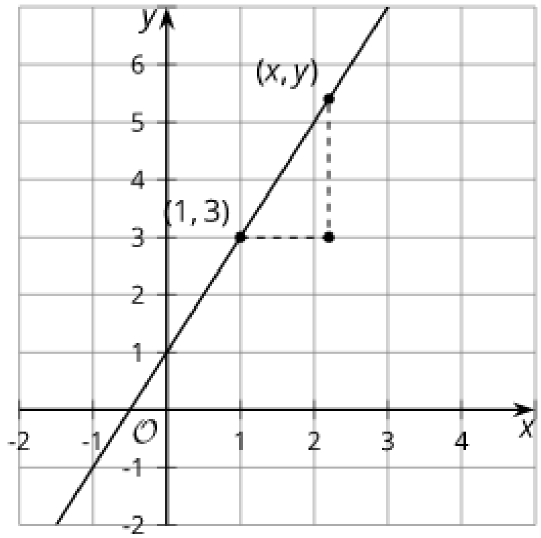
   a. $(1, 6)$ and $(3, 8)$

   b. $(11, 6)$ and $(13, 9)$

   c. $(-4, 5)$ and $(-6, -1)$

   d. $(3, 7)$ and $(10, 0)$

2. The image shows a line.



a. Find the slope between the two points $(1,3)$ and $(3,7)$ using the formula for slope.

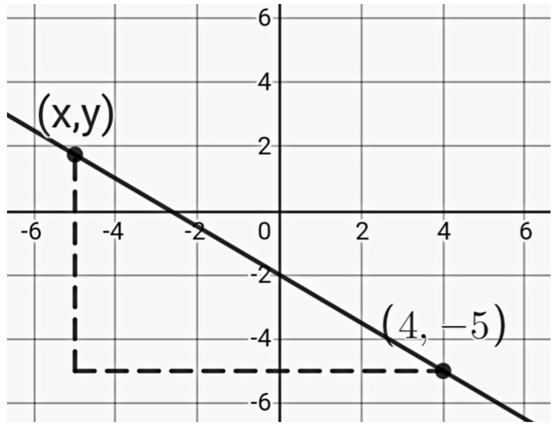
b. Using the same formula as above, write an equation that says the slope between the points $(1,3)$ and (x,y) is 2.

c. Look at this equation: $y-3=2(x-1)$

How does it relate to the equation you wrote?

d. How can you quickly tell from looking at this equation that $(1,3)$ must be on the line?

3. The image shows a line.



a. Find the slope between the two points $(4, -5)$ and $(0, -2)$ using the formula for slope.

b. Using the same formula as above, write an equation that says the slope between the points $(4, -5)$ and (x, y) is $-\frac{3}{4}$.

c. Look at this equation: $y + 5 = -\frac{3}{4}(x - 4)$.
How does it relate to the equation you wrote?

d. How can you quickly tell from looking at this equation that $(4, -5)$ must be on the line?

4. Here is an equation for another line: $y - 7 = \frac{1}{2}(x - 5)$
   a. What point do you know this line passes through?
   b. What is the slope of this line?

5. Next, let's write a general equation that we can use for any line. Suppose we know a line passes through a particular point (h, k).
   a. Write an equation that says the slope between point (x, y) and (h, k) is m.
   b. Look at this equation: $y - k = m(x - h)$. How does it relate to the equation you wrote?

Activity 2: Using Point-Slope Form

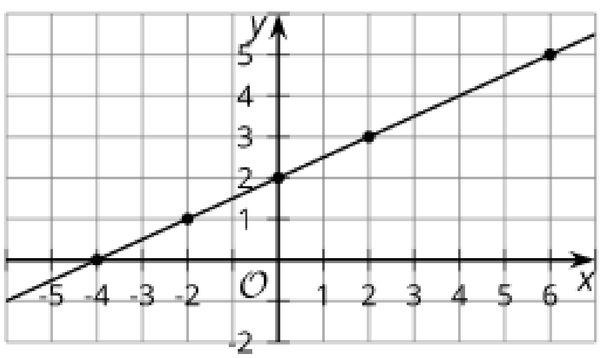
1. Write an equation that describes each line.

   a. the line passing through point $(-2, 8)$ with slope $\frac{4}{5}$

   b. the line passing through point $(0, 7)$ with slope $-\frac{7}{3}$

   c. the line passing through point $(\frac{1}{2}, 0)$ with slope -1

   d. the line in the image:



2. Using the structure of the equation, what point do you know each line passes through? What's the line's slope?

   a. $y - 5 = \frac{3}{2}(x + 4)$

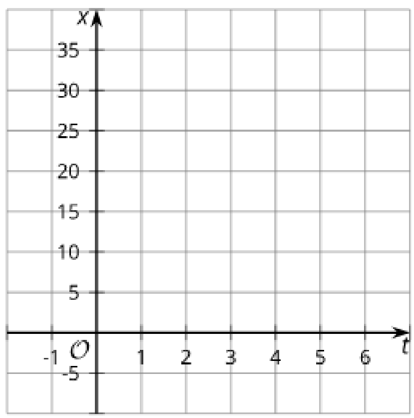
   b. $y + 2 = 5x$

   c. $y = -2(x - \frac{5}{8})$

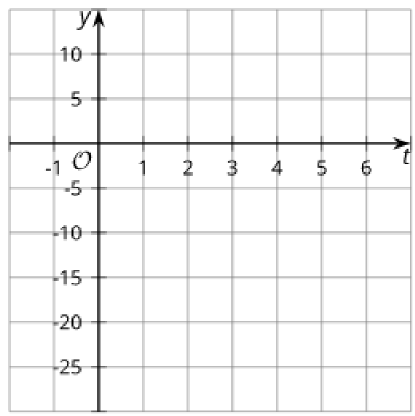
Are You Ready For More?

Another way to describe a line, or other graphs, is to think about the coordinates as changing over time. This is especially helpful if we're thinking about tracing an object's movement. This example describes the x- and y-coordinates separately, each in terms of time, t.

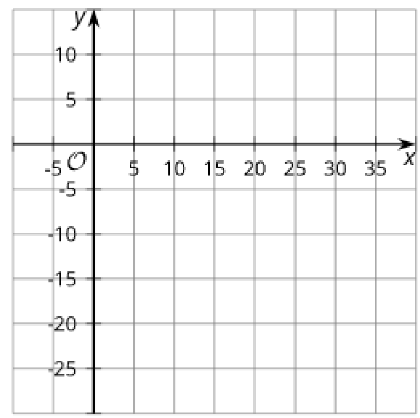
Grid A



Grid B



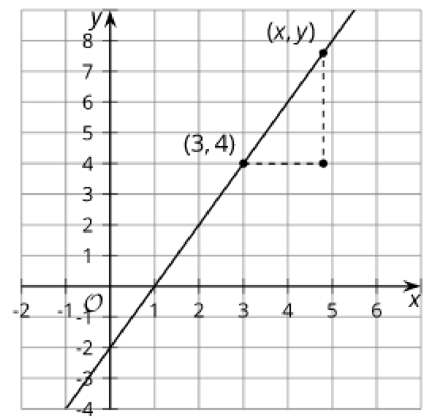
Grid C



1. On grid A, create a graph of $x = 2 + 5t$ for $-2 \le t \le 7$ with x on the vertical axis and t on the horizontal axis.
2. On grid B, create a graph of $y = 3 – 4t$ for $-2 \le t \le 7$ with y on the vertical axis and t on the horizontal axis.
3. On grid C, create a graph of the set of points $(2 + 5t, 3 – 4t)$ for $-2 \le t \le 7$ on the xy-plane.

Lesson Debrief

Lesson 4 Summary and Glossary



The line in the image can be defined as the set of points that make a slope of 2 with the point $(3,4)$. The equation $\frac{y-4}{x-3}=2$ says the slope between points (x,y) and $(3,4)$ is 2. This equation can be rearranged to look like $y-4=2(x-3)$.

The equation is now in **point-slope form**, or $y-k=m(x-h)$, where:

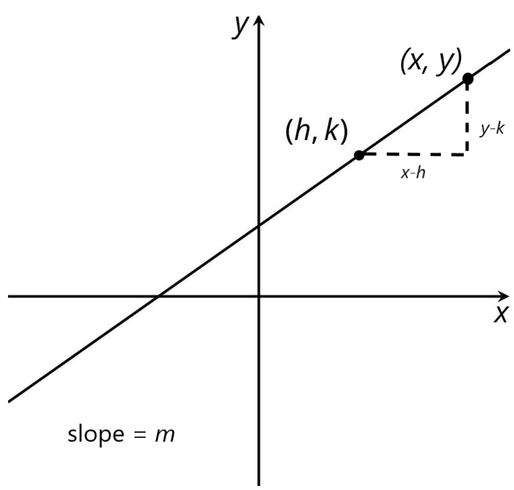
- (x,y) is any point on the line
- (h,k) is a particular point on the line that we choose to substitute into the equation
- m is the slope of the line

Other ways to write the equation of a line include slope-intercept form, $y=mx+b$, and standard form, $Ax+By=C$.

To write the equation of a line passing through $(3,1)$ and $(0,5)$, start by finding the slope of the line. The slope is $-\frac{4}{3}$ because $\frac{5-1}{0-3}=-\frac{4}{3}$. Substitute this value for m to get $y-k=-\frac{4}{3}(x-h)$. Now we can choose any known point on the line to substitute for (h,k). If we choose $(3,1)$, we can write the equation of the line as $y-1=-\frac{4}{3}(x-3)$.

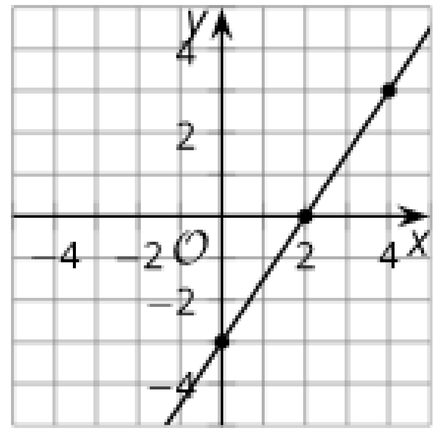
We could also use $(0,5)$ as the point, giving $y-5=-\frac{4}{3}(x-0)$. We can rearrange the equation to see how point-slope and slope-intercept forms relate, getting $y=-\frac{4}{3}x+5$. Notice $(0,5)$ is the y-intercept of the line. The graphs of all 3 of these equations look the same.

Point-slope form: The form of a linear equation written as $y-k=m(x-h)$, where m is the slope of the line and (h,k) is a point on the line. Point-slope form can also be written as $y=k+m(x-h)$.



Unit 3 Lesson 4 Practice Problems

1. Select **all** the equations that represent the graph shown.



   a. $3x - 2y = 6$
   b. $y = \frac{3}{2}x + 3$
   c. $y = \frac{3}{2}x - 3$
   d. $y - 3 = \frac{3}{2}(x - 4)$
   e. $y - 6 = \frac{3}{2}(x - 2)$

2. Write the equation $y + 2 = 3(x + 1)$ in slope-intercept form.

3. A line with slope $\frac{3}{2}$ passes through the point $(1, 3)$.

   a. Explain why $(3, 6)$ is on this line.

   b. Explain why $(0, 0)$ is not on this line.

   c. Is the point $(13, 22)$ on this line? Explain why or why not.

4. Write an equation of the line that passes through $(1, 3)$ and has a slope of $\frac{5}{4}$.

5. Write two equivalent equations for a line with x-intercept $(3,0)$ and y-intercept $(0,2)$.

6. Clare has been working to save money and wants to have an equation to model the amount of money in her bank account.

   She has been depositing $175 a month consistently. She doesn't remember how much money she deposited initially; however, on her last statement she saw that her account has been open for 10 months and currently has $2475 in it.

   Write an equation for the amount of money in Clare's bank account after x months. Which equation form did you choose?[1]

[1] Adapted from Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

7. Solve for the indicated variable in each part.[2]

a. $x : ax = 7$

b. $p : 8 + p = w$

c. $y : \frac{1}{2}y = k$

d. $x : y = mx + b$

(From Unit 2)

8. Elena's mother's painting service charges $10 per job and $0.20 per square foot. If she earned $50 for painting one job, how many square feet did she paint at the job? Write an equation and solve.[3]

(From Unit 2)

[2] Adapted from Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)
[3] Adapted from Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org (see above).

9. Han took a math test with 20 questions, and each question is worth an equal number of points. The test is worth 100 points total.[4]

   a. Write an equation that can be used to calculate Han's score based on the number of questions he got right on the test.

   b. If a score of 70 points earns a grade of a C, how many questions would Han need to get right to get at least a C on the test?

   c. If a score of 80 points earns a grade of B, how many questions would Han need to get at least a B on the test?

   d. Suppose Han got a score of 60% and then chose to retake the test. On the retake, Han got all the questions right that he got right the first time, and he also got half the questions right that he got wrong the first time. What percent of the questions did Han get right, in total, on the retake?

(From Unit 2)

[4] Adapted from Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

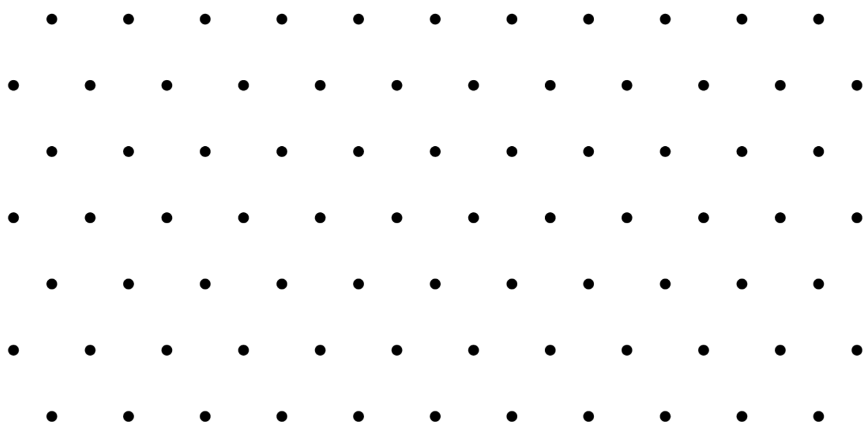
Lesson 5: Equations of Parallel Lines

Learning Targets

- I can determine if two lines are parallel.
- I can write the equation of a line parallel to a given line that passes through a given point.

Bridge

Here is a field of dots. Each dot represents a point.[1]



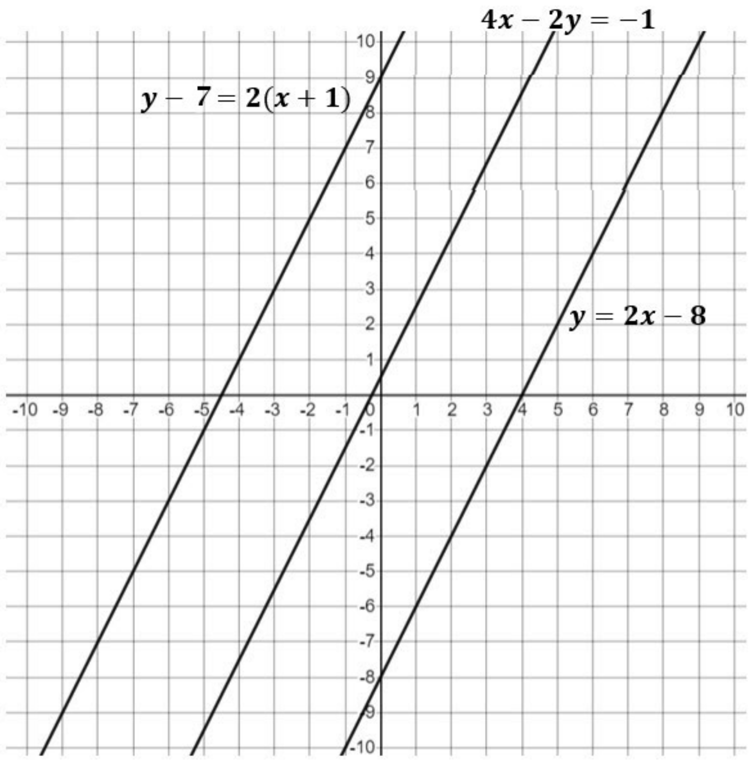
1. Draw a line through at least two points. Label it line h.
2. Draw another line that goes through at least two points and intersects your first line. Label it line g.
3. Can you draw a new line that you think would never intersect:
    a. line h?
    b. line g?

   If so, draw the line. Be prepared to explain or show how you know the lines would never cross. If not, explain or show why it can't be done.

[1] Adapted from IM K–5 https://curriculum.illustrativemathematics.org/K5/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

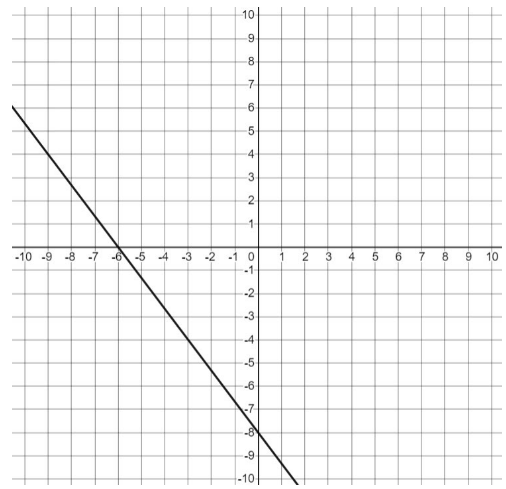
Warm-up: Three Lines

The lines of three linear equations are graphed below. What do you notice? What do you wonder?



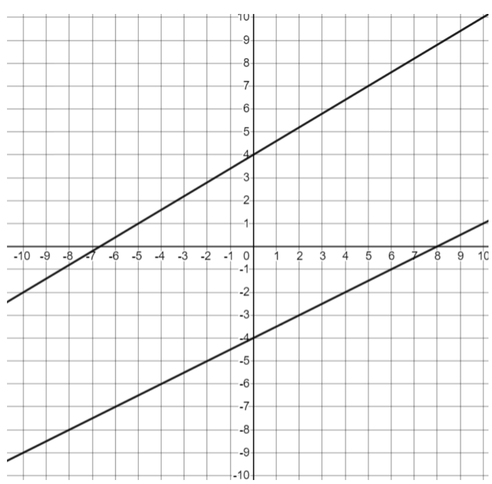
Activity 1: Graphing Parallel Lines

1. Given this graph of a linear equation, graph a line that is parallel. Be prepared to explain your process.



2. For each of the following pairs of lines, determine if the lines are parallel or not. Provide justification for your response.

a.

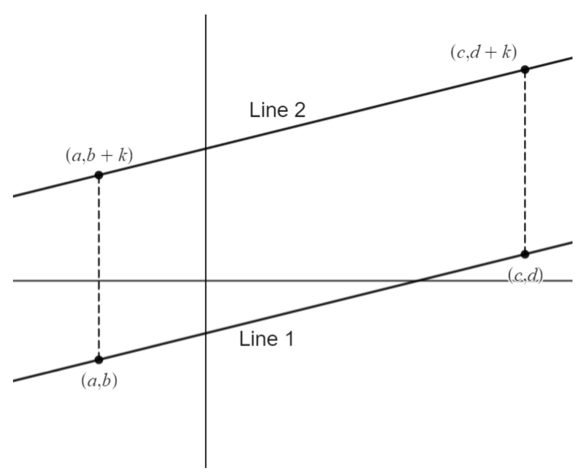


b. $y = 3x - 7$

$6x - 2y = -8$

Are You Ready For More?

The following statements relate to the graph:



- Points (a,b) and $(c.d)$ are on Line 1.
- Each point was translated k units up.
- Points $(a,b+k)$ and $(c,d+k)$ are on Line 2.

Prove that the slopes of the lines are the same.

Activity 2: Writing Equations of Parallel Lines

1. Write the equation of a line that is parallel to $y = 5x + 4$ and passes through the point $(3, -2)$.

2. Write the equation of a line that is parallel to $2x + 3y = 4$ and passes through the point $(3, 1)$.

Lesson Debrief

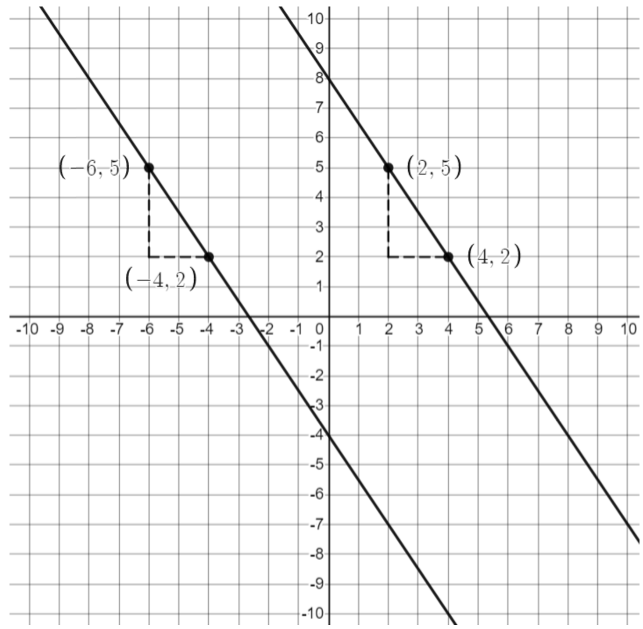
Lesson 5 Summary and Glossary

Two lines on a coordinate plane are parallel if and only if they have the same slope. This means:

- If two lines are parallel, then their slopes are the same.
- If the slopes of two lines are the same, then the lines are parallel.

To determine if two lines are parallel, compare the slopes of the lines.

- In the graph, slope triangles are used to identify the slope. The two lines graphed have a slope of $\frac{-3}{2}$, so they are parallel.
- Given two linear equations, $y = 2x + 9$ and $8x - 4y = 12$:
  - The slope of the line defined by the first equation is 2.
  - To find the slope of the second equation, first rewrite it as as $y = 2x - 3$.
  - The slope of the line defined by the second equation is 2.
  - The slopes of each line are the same so the lines are parallel.

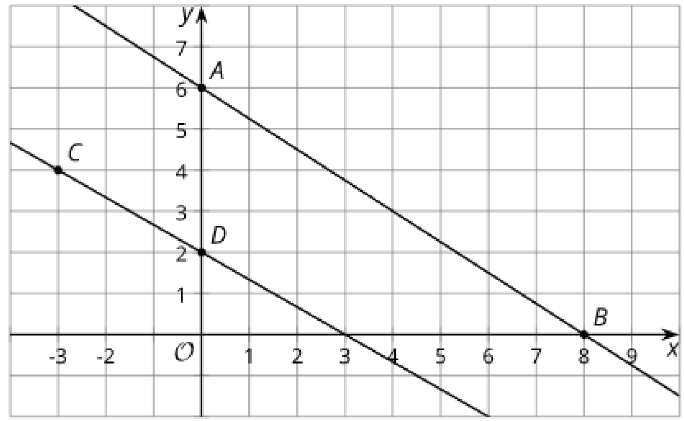


To write the equation of a line that is parallel to the line $y = \frac{4}{5}x - 7$ and passes through a point $(15, 2)$, start by identifying the slope. The slope of the line is $\frac{4}{5}$. Using point-slope form, the equation of the parallel line is $y - 2 = \frac{4}{5}(x - 15)$ which can be rewritten as $y = \frac{4}{5}x - 10$.

Unit 3 Lesson 5 Practice Problems[2]

1. The image shows lines AB and CD.

   Are the two lines parallel? Explain or show your reasoning.



2. Select **all** equations that are parallel to the line $2x + 5y = 8$.

   a. $y = \frac{2}{5}x + 4$

   b. $y = -\frac{2}{5}x + 4$

   c. $y - 2 = \frac{2}{5}(x + 1)$

   d. $y - 2 = -\frac{2}{5}(x - 1)$

   e. $10x + 25y = 40$

[2] Adapted from IM 9–12 Math https://curriculum.illustrativemathematics.org/HS/index.html,

3. Write an equation of a line that passes through $(-1, 2)$ and is parallel to a line with x-intercept $(3, 0)$ and y-intercept $(0, 1)$.

4. Write an equation of the line with slope $\frac{2}{3}$ that goes through the point $(-2, 5)$.

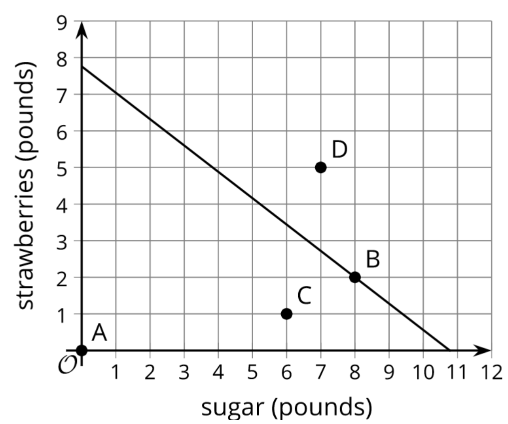
(From Unit 3, Lesson 4)

5. Priya and Han each wrote an equation of a line with slope $\frac{1}{3}$ that passes through the point $(1, 2)$. Priya's equation is $y - 2 = \frac{1}{3}(x - 1)$ and Han's equation is $3y - x = 5$. Do you agree with either of them? Explain or show your reasoning.

(From Unit 3, Lesson 4)

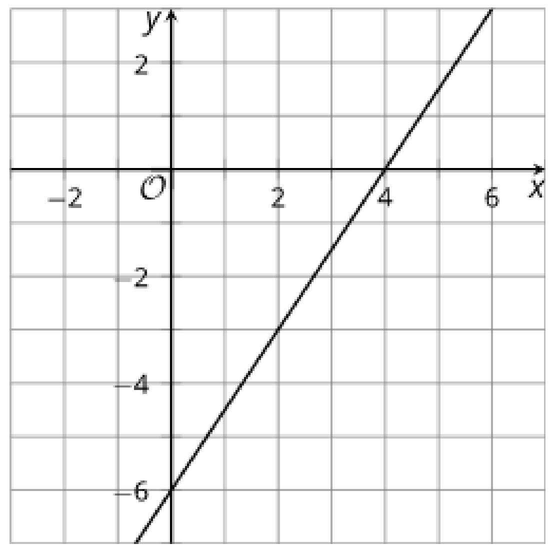
6. Jada brought some sugar and strawberries to make strawberry jam. Sugar costs $1.80 per pound, and strawberries cost $2.50 per pound. Jada spent a total of $19.40.

Which point on the coordinate plane could represent the pounds of sugar and strawberries that Jada used to make jam?



(From Unit 3, Lesson 1)

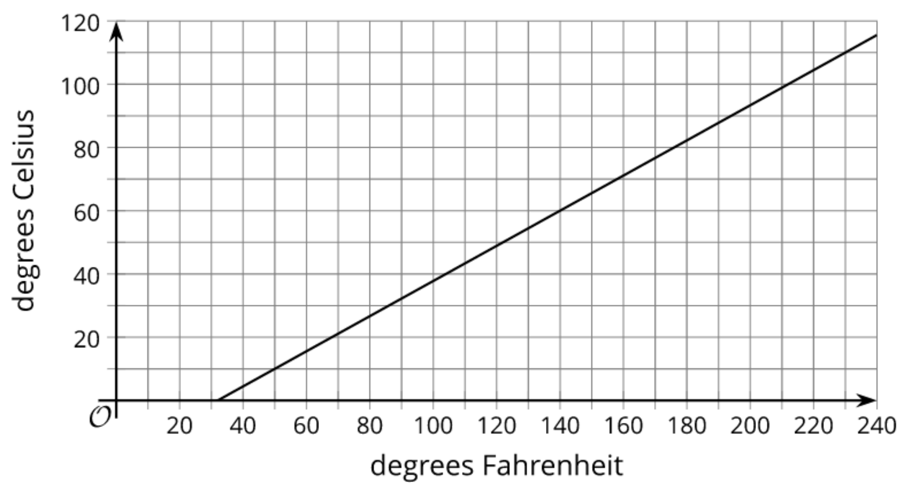
7. Here is a graph of the equation $3x - 2y = 12$. Select **all** coordinate pairs that represent a solution to the equation.



   a. $(2, -3)$
   b. $(4, 0)$
   c. $(5, -1)$
   d. $(0, -6)$
   e. $(2, 3)$

(From Unit 3, Lesson 1)

8. The graph shows the relationship between temperature in degrees Celsius and temperature in degrees Fahrenheit.



   a. Mark the point on the graph that shows the temperature in Celsius when it is 60 degrees Fahrenheit.
   b. Mark the point on the graph that shows the temperature in Fahrenheit when it is 60 degrees Celsius.
   c. Water boils at 100 degrees Celsius. Use the graph to approximate the boiling temperature in Fahrenheit, or to confirm it, if you know what it is.
   d. The equation that converts Fahrenheit to Celsius is $C = \frac{5}{9}(F - 32)$. Use it to calculate the temperature in Celsius when it is 60 degrees Fahrenheit. (This answer will be more exact than the point you found in the first part.)

(From Unit 3, Lesson 1)

9. A cell phone company offers a plan that costs $35.99 and includes unlimited texting. Another company offers a plan that costs $19.99 and charges $0.25 per text. For what number of texts does the second company's plan cost more than the first company's plan?[3]

    a. Write and solve an inequality that models this situation. Be sure to define your variables.

    b. Describe in words the quantities that would work in this situation.

(From Unit 2)

[3] Adapted from Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

Lesson 6: Equations of Perpendicular Lines

Learning Targets

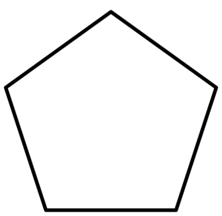
- I can determine if two lines are perpendicular.
- I can write the equation of a line perpendicular to a given line that passes through a given point.

Bridge

Which shapes have sides that are perpendicular to one another?[1]

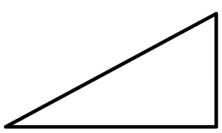
Mark the perpendicular sides. How would you explain the word “perpendicular” to a sixth grader?

A

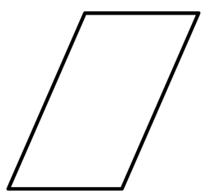


B

C



D



Warm-up: Special Products

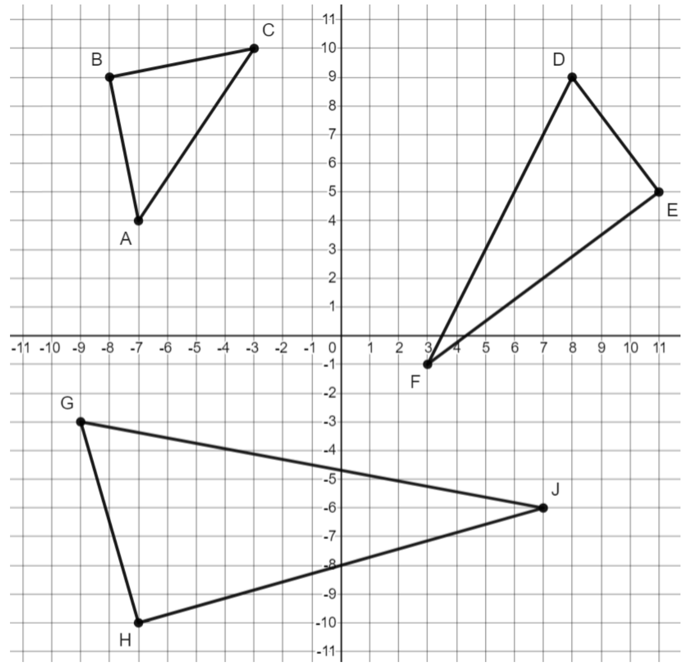
Which one doesn’t belong? Explain your reasoning.

a. $8 \times \frac{1}{8}$	b. $\frac{2}{3} \times \frac{3}{2}$
c. $-\frac{4}{7} \times -\frac{7}{4}$	d. $\frac{5}{6} \times -\frac{6}{5}$

[1] Adapted from IM K–5 https://curriculum.illustrativemathematics.org/K5/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

Activity 1: Make a Conjecture

The image shows three right triangles where angles B, E, and H are right angles.



1. Complete the tables with the slopes of the line segments that form each right angle.

Triangle ABC		
Slope of side AB	**Slope of side BC**	**Product**

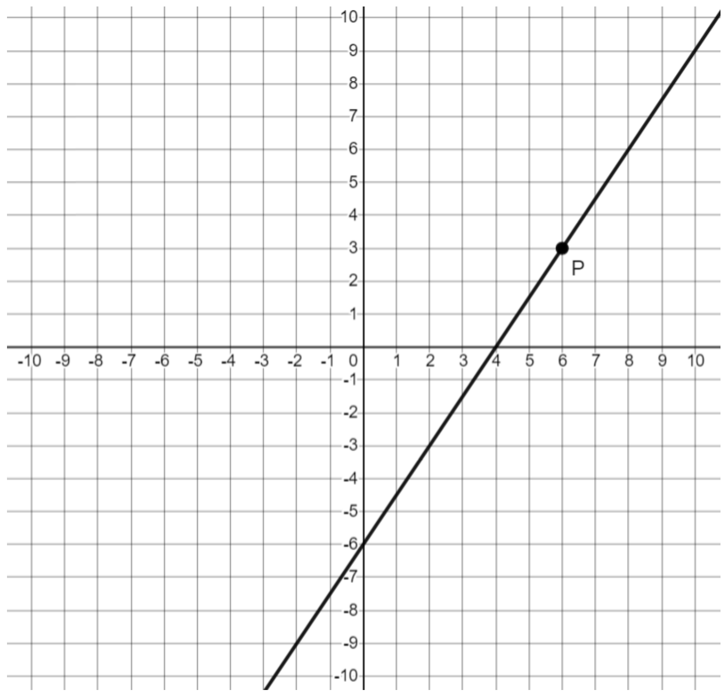
Triangle DEF		
Slope of side DE	**Slope of side EF**	**Product**

Triangle GHJ		
Slope of side GH	**Slope of side HJ**	**Product**

2. Use your slope calculations to make a conjecture about slopes of perpendicular lines.

Activity 2: Writing Equations of Perpendicular Lines

1. Write the equation of the line perpendicular to the given line that passes through the point P.



2. Kiran tried to write an equation for the line perpendicular to $y = -3x + 2$ that passes through the point $(-4, 2)$. His answer was $y - 2 = 3(x + 4)$.

    a. The line is not perpendicular. Explain what Kiran's mistake was.

    b. What is the correct equation of the line?

Are You Ready For More?

1. Line l is represented by the equation $y = \frac{2}{3}x + 3$. Write an equation of the line perpendicular to l, passing through $(-6, 4)$. Call this line p.

2. Write an equation of the line perpendicular to p, passing through $(3, -2)$. Call this line n.

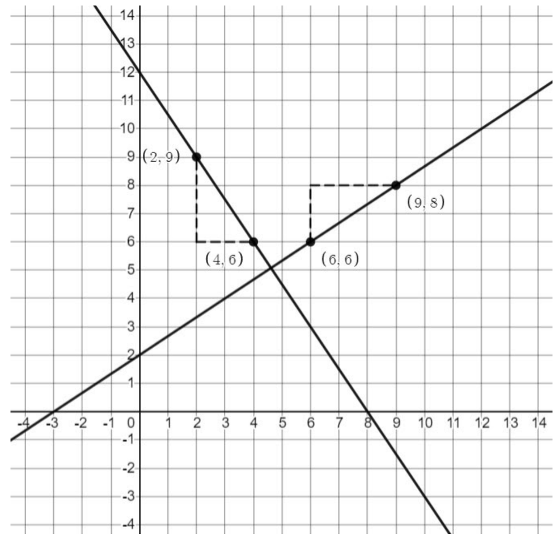
3. What do you notice about lines l and n? Does this always happen? Show or explain your work.

Lesson Debrief

Lesson 6 Summary and Glossary

Two lines on a coordinate plane are perpendicular if and only if their slopes are opposite reciprocals. This means:

- If two lines are perpendicular then their slopes are opposite reciprocals.
- If the slopes of two lines are opposite reciprocals then the lines are perpendicular.



Opposite reciprocals: A pair of numbers that can be expressed as $\frac{a}{b}$ and $-\frac{b}{a}$ where $a \neq 0$ and $b \neq 0$. Opposite reciprocals have a product of -1.

To determine if two lines are perpendicular, compare the slopes of the lines.

- In the graph, slope triangles are used to identify the slope. One line has a slope of $\frac{-3}{2}$ and the other has a slope of $\frac{2}{3}$. The slopes are opposite reciprocals so the lines are perpendicular.
- Given two linear equations, $y = 5x + 1$ and $2x + 10y = -20$:
  - The slope of the line defined by the first equation is 5, which can be written as $\frac{5}{1}$.
  - To find the slope of the second equation, first rewrite it as as $y = -\frac{1}{5}x - 2$.
  - The slope of the line defined by the second equation is $-\frac{1}{5}$.
  - The slopes of $\frac{5}{1}$ and $-\frac{1}{5}$ are opposite reciprocals so the lines are perpendicular.

To write the equation of a line that is perpendicular to the line $y = \frac{3}{8}x + 2$ and passes through the point $(-6, 1)$, start by identifying the slope. The slope of the line is $\frac{3}{8}$. The slope of the perpendicular line is the opposite reciprocal $-\frac{8}{3}$. Using point-slope form, the equation of the perpendicular line is $y - 1 = -\frac{8}{3}(x + 6)$, which can be rewritten as $y = -\frac{8}{3}x - 15$.

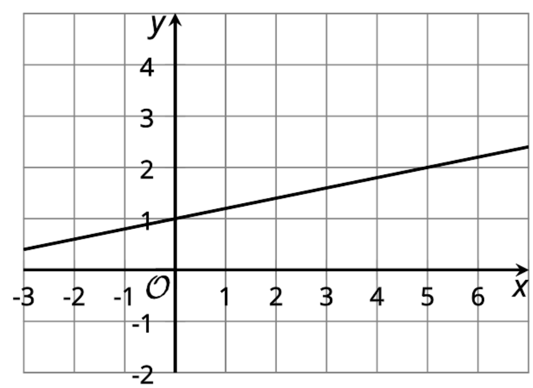
Unit 3 Lesson 6 Practice Problems

1. Write an equation for a line that passes through the origin and is perpendicular to $y = 5x - 2$.

2. Match each line with a perpendicular line.

a. $y = 5x + 2$	1. the line through $(2, 12)$ and $(17, 9)$
b. $y - 2.25 = -2(x - 2)$	2. $y = -\frac{1}{2}x + 5$
c. the line through $(-1, 5)$ and $(1, 9)$	3. $2x - 4y = 10$

3. For each equation, is the graph of the equation parallel to the line shown, perpendicular to the line shown, or neither? Put a checkmark in the appropriate column.[2]



Equation	Parallel	Perpendicular	Neither
a. $y = 0.2x$			
b. $y = -2x + 1$			
c. $y = 5x - 3$			
d. $(y-3) = -5(x-4)$			
e. $(y-1) = 2(x-3)$			
f. $5x + y = 3$			

[2] Adapted from IM 9–12 Math Geometry https://curriculum.illustrativemathematics.org/HS/index.html,

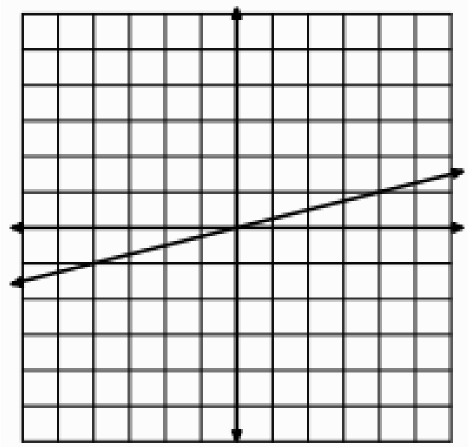
4. Line p has an x-intercept at $(3,0)$ and y-intercept at $(0,-4)$.

    a. Write the equation of the line p.

    b. Write the equation of the line parallel to line p that passes through $(2,5)$.

(From Unit 3, Lesson 5)

5. The graph at the right shows the line $y=\frac{1}{4}x$.



    a. On the same grid, graph a parallel line that is two units below it.

    b. Write the equation of the new line in slope-intercept form.

    c. Write the y-intercept of the new line as an ordered pair.

    d. Write the x-intercept of the new line as an ordered pair.

    e. Write the equation of the new line in point-slope form using the y-intercept.

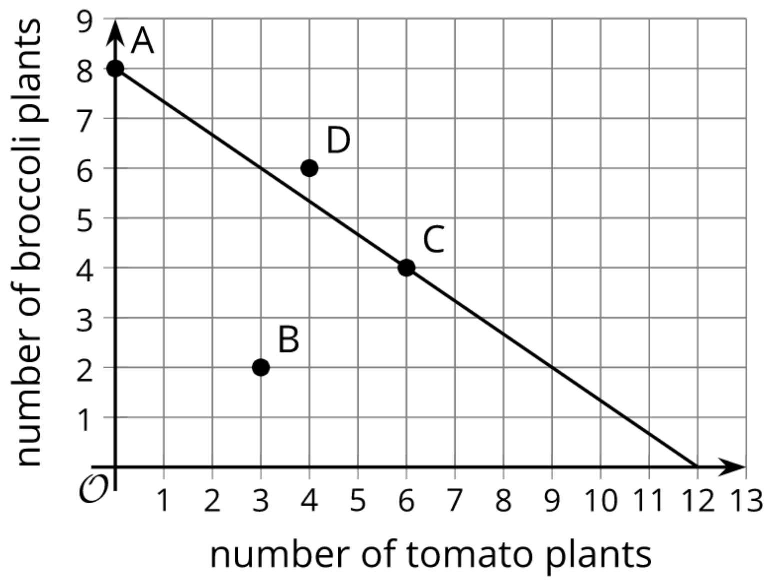
    f. Write the equation of the new line in point-slope form using the x-intercept.

    g. Explain in what ways the equations in parts b, e, and f are the same and in what way they are different.

(From Unit 3, Lessons 4–5)[3]

[3] Adapted from Geometry Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

6. To grow properly, each tomato plant needs 1.5 square feet of soil and each broccoli plant needs 2.25 square feet of soil. The graph shows the different combinations of broccoli and tomato plants in an 18 square foot plot of soil.[4]



Match each point to the statement that describes it.

a. Point A	1. The soil is fully used when six tomato plants and four broccoli plants are planted.
b. Point B	2. Only broccoli was planted, but the plot is fully used and all plants can grow properly.
c. Point C	3. After three tomato plants and two broccoli plants were planted, there is still extra space in the plot.
d. Point D	4. With four tomato plants and six broccoli plants planted, the plot is overcrowded.

(From Unit 3, Lesson 1)

[4] Adapted from IM 9–12 Math Algebra 1 https://curriculum.illustrativemathematics.org/HS/index.html,

7. A catering company is setting up for a wedding. They expect 150 people to attend. They can provide small tables that seat 6 people and large tables that seat 10 people.[5]

    a. Find a combination of small and large tables that seats exactly 150 people.

    b. Let x represent the number of small tables and y represent the number of large tables. Write an equation to represent the relationship between x and y.

    c. Explain what the point $(20,5)$ means in this situation.

    d. Is the point $(20,5)$ a solution to the equation you wrote? Explain your reasoning.

(From Unit 3 Lesson 1)

[5] Adapted from IM 9–12 Math Algebra 1 https://curriculum.illustrativemathematics.org/HS/index.html,

8. Indicate if the following statements are true or false. Explain your thinking.[6]

   a. The notation $12 < x$ means the same thing as $x < 12$. It works just like $12 = x$ and $x = 12$.

   b. The inequality $-2(x + 10) \geq 75$ says the same thing as $-2x - 20 \geq 75$. I can multiply by -2 on the left side without reversing the inequality symbol.

   c. When solving the inequality $10x + 22 < 2$, the second step should say $10x > -20$ because I added -22 to both sides and I got a negative number on the right.

   d. When solving the inequality $-5x \geq 45$, the answer is $x \leq -9$.

   e. The words that describe the inequality $x \geq 100$ are "x is greater than or equal to 100."

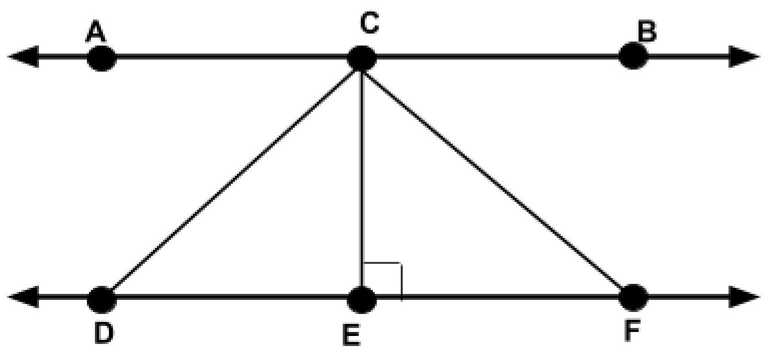
(From Unit 2)

[6] Adapted from Algebra 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

9. Solve the multi-step inequality: $\frac{3(x-4)}{12} \leq \frac{2x}{3}$

(From Unit 2)

10.
   a. Which segment below is perpendicular to line AB? How do you know?



   b. Which segment has the shortest length: CD, CE, or CF? How do you know?

(Addressing NC.4.G.1)

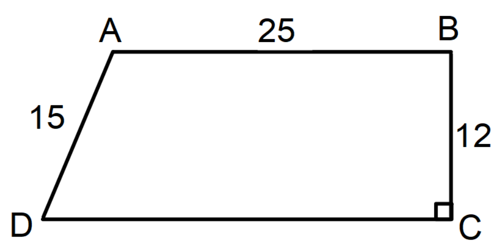
Lesson 7: Perimeter and Area of Shapes in the Coordinate Plane

Learning Targets

- I can find the length of a side of a polygon in the coordinate plane.
- I can find the perimeter and area of triangles and quadrilaterals in the coordinate plane.
- I can determine if a quadrilateral in the coordinate plane is a parallelogram, rectangle, rhombus, or square.

Bridge

Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 25$, and $BC = 12$.

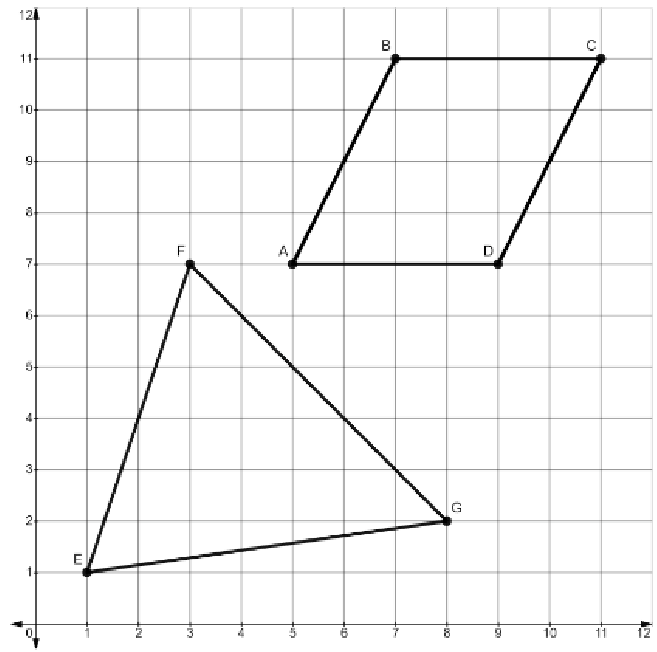


1. What is the area of the trapezoid?

2. What is the perimeter of the trapezoid?

Warm-up: How Do You Know?

1. Is quadrilateral $ABCD$ a parallelogram? Explain or show your reasoning.



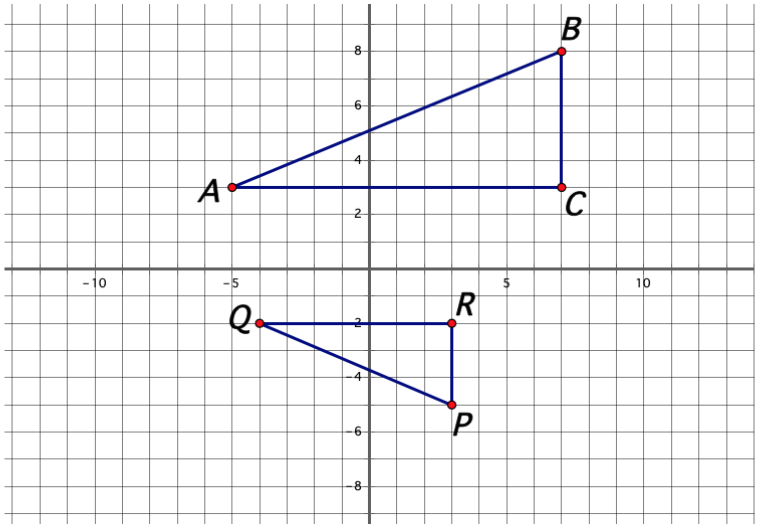
2. What is the area of $ABCD$? Explain or show your reasoning.

3. What is the perimeter of $ABCD$? Round to the nearest hundredth. Explain or show your reasoning.

4. Is triangle EFG isosceles? Explain or show your reasoning.

Activity 1: Finding the Distance

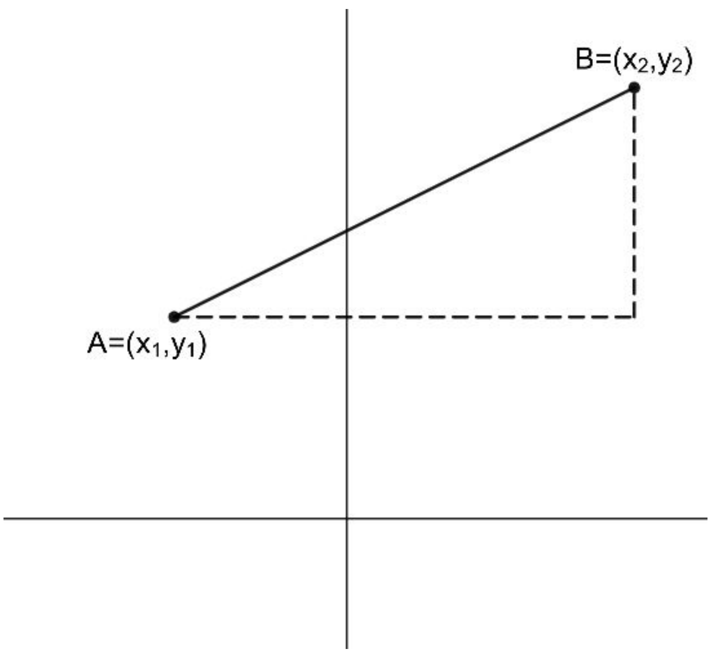
1. Below is a picture of two triangles with vertices on coordinate grid points[1]:



What are the perimeters of $\Delta\, ABC$ and $\Delta\, PQR$?

2. Here is a line segment that can stand for any line segment. Because its coordinates can be any values, we can call the coordinates of *A* (x_1, y_1) and the coordinates of *B* (x_2, y_2).

   Find the lengths of the two dashed lines in terms of those coordinates.



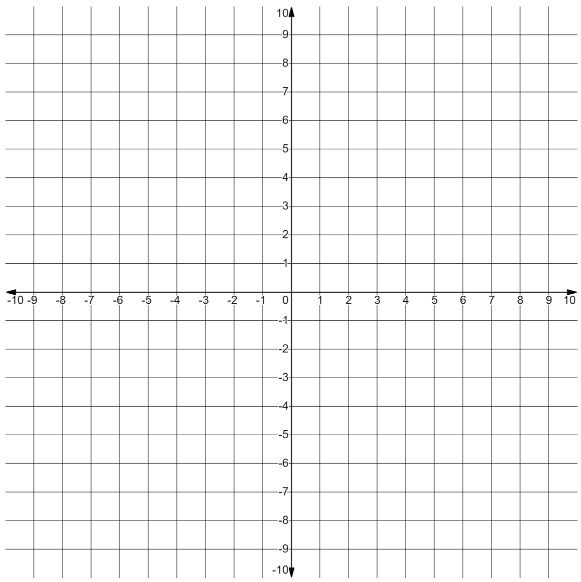
Are You Ready For More?

Using the same graph above, find the length of AB in terms of x_1, x_2, y_1, and y_2.

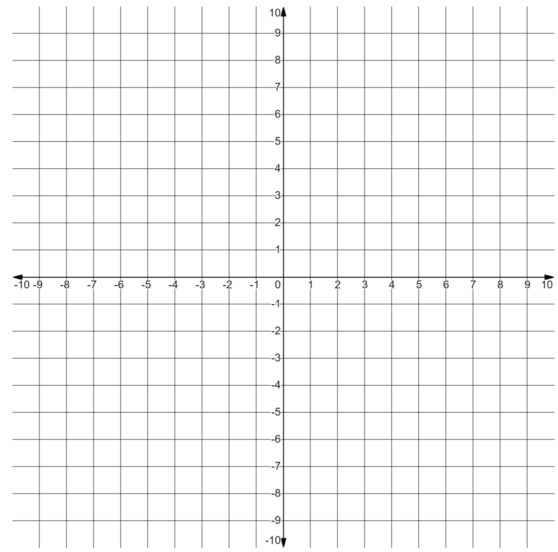
[1] Adapted from https://tasks.illustrativemathematics.org/

Activity 2: What Shape Am I?

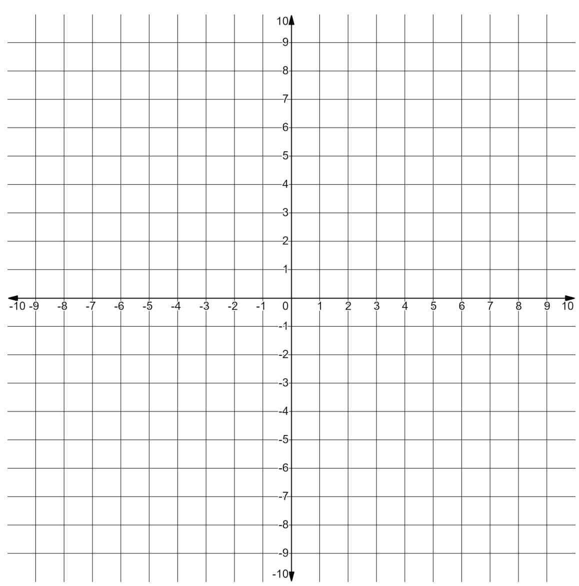
1. A quadrilateral has vertices $A(1,0)$, $B(5,-2)$, $C(7,2)$, and $D(3,4)$. Is $ABCD$ a square?



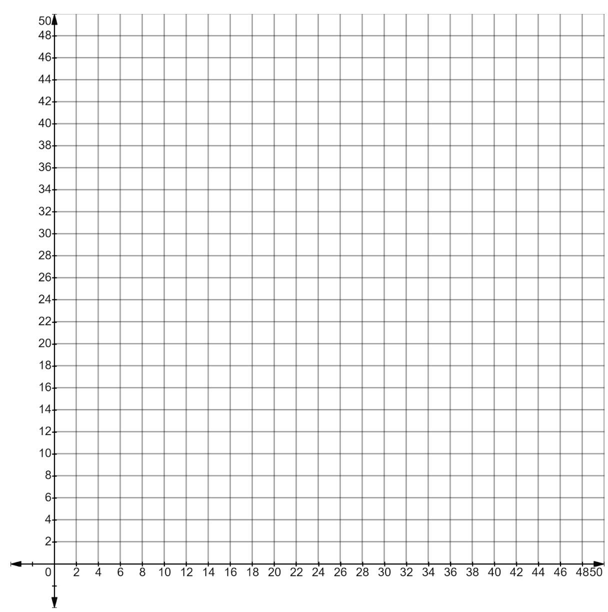
2. A quadrilateral has vertices $E(-2,3)$, $F(-5,11)$, $G(1,9)$, and $H(4,1)$. Is $EFGH$ a parallelogram?



3. A quadrilateral has vertices $K(-5,-1)$, $L(-1,-4)$, $M(5,4)$, and $N(1,7)$. Is $KLMN$ a rectangle?



4. A quadrilateral has vertices $Q(1,10)$, $R(26,33)$, $S(50,13)$, $T(26,20)$. Is $QRST$ a rhombus?



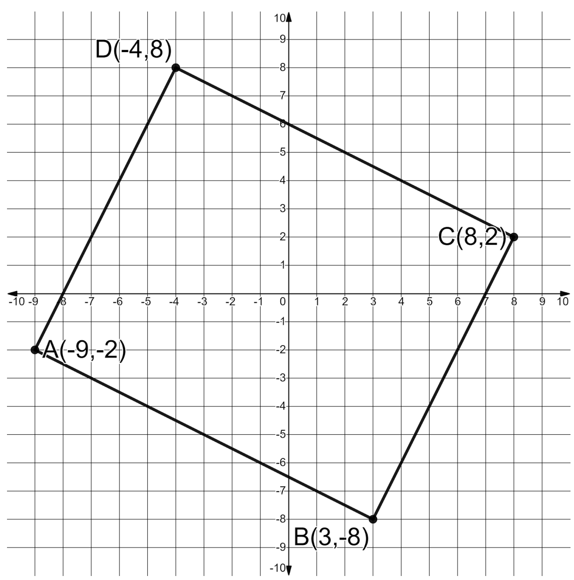
5. Find the area of the quadrilateral in question 3.

Lesson Debrief

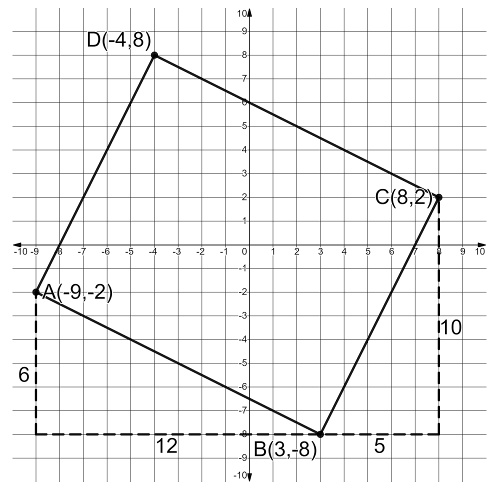
Lesson 7 Summary and Glossary

Take a look at quadrilateral $ABCD$. Is it a square? Could be, but it looks a little squished. A rectangle? Maybe the corners aren't perfect right angles. Or maybe a parallelogram? Or none of these shapes?

We can use ideas about distance, parallel lines, and perpendicular lines to find out.



To be a square, $ABCD$ would have to have all of its sides the same length, as well as having all 90-degree angles. Let's start by checking the side lengths. To find the lengths of AB and BC, draw in extra lines so that we can use the Pythagorean Theorem:



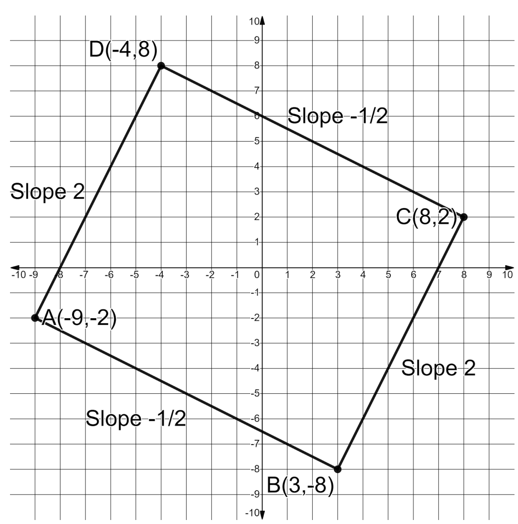
$$AB = \sqrt{6^2 + 12^2} \qquad BC = \sqrt{5^2 + 10^2}$$
$$AB = \sqrt{180} \approx 13.42 \qquad BC = \sqrt{125} \approx 11.18$$

Since AB and BC are different lengths, $ABCD$ can't be a square. But it could still be a rectangle. Let's check the slopes now to see if the sides are perpendicular. We've already made slope triangles for sides AB and BC, so let's start there.

Slope of AB: $-\frac{6}{12} = -\frac{1}{2}$, Slope of BC: $\frac{10}{5} = \frac{2}{1}$

So far so good: $-\frac{1}{2}$ and 2 are opposite reciprocals. But that only tells us that angle B is a right angle; we don't know about the others. We should find the slopes of the other sides. $-\frac{1}{2}$ and 2 are opposite reciprocals. But that only tells us that angle B is a right angle; we don't know about the others. We should find the slopes of the other sides.

Slope of CD: $\frac{2-8}{8-(-4)} = -\frac{6}{12} = -\frac{1}{2}$, Slope of DA: $\frac{8-(-2)}{-4-(-9)} = \frac{10}{5} = \frac{2}{1}$



Since each angle is formed by a pair of line segments, one with slope $\frac{1}{2}$ and the other with slope 2, all four angles are right angles. We can now say for sure that $\frac{1}{2}$ and the other with slope 2 $ABCD$ is a rectangle.

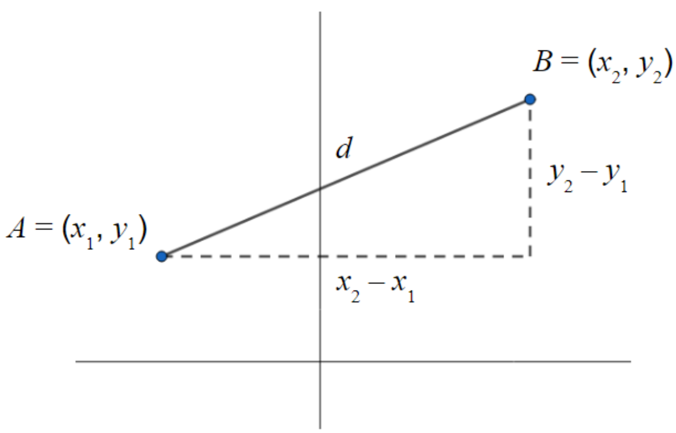
This also shows that $ABCD$ must be a parallelogram. Each pair of opposite sides has the same slope, which means that each pair of opposite sides is parallel.

Now that we know that $ABCD$ is a rectangle, we can find its area and perimeter.

We've already found two of its side lengths, and since it is a rectangle the other two side lengths must match. So the perimeter of $ABCD$ is $2 \cdot \sqrt{180} + 2 \cdot \sqrt{125} \approx 49.19$.

To find the area of $ABCD$, we can multiply the length times the width: $\sqrt{180} \cdot \sqrt{125} \approx 150$.

When we calculated the side lengths AB and CD, we made a triangle with horizontal and vertical sides and used the Pythagorean theorem to find the length of the hypotenuse. Here is a more general picture of what we did:



To find the length of the horizontal sides, subtract the x-coordinates. To find the length of the vertical sides, subtract the y-coordinates. When we use the Pythagorean theorem on these side lengths, we get the **distance formula**.

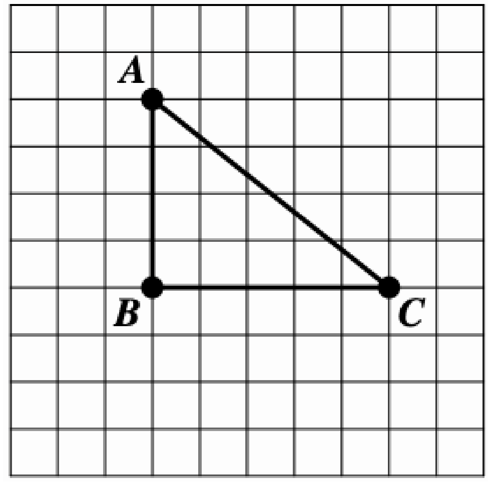
Distance formula: The distance d between points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

When dealing with small numbers, you will never need to use the distance formula as long as you remember to use the Pythagorean theorem to find lengths. However, the distance formula will come up in more advanced courses.

Unit 3 Lesson 7 Practice Problems

1. Use the graph.[2]



   a. Find AB.

   b. Find BC.

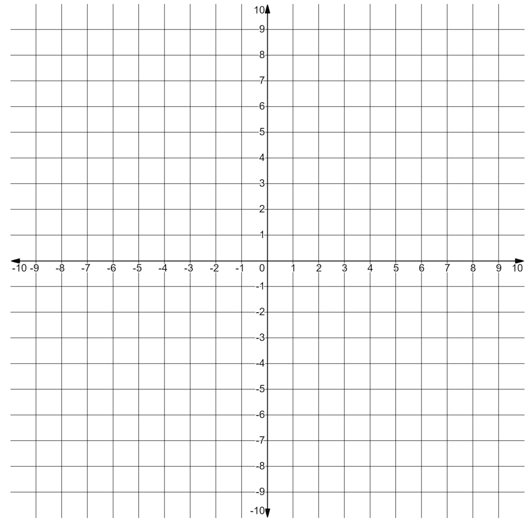
   c. Find AC.

   d. Why is it easier to find the distance between point A and point B and point C than it is to find the distance between point A and point C?

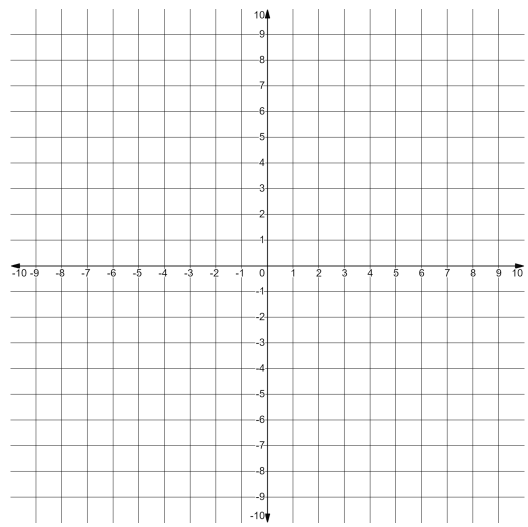
   e. Explain how to find the distance between point A and point C.

[2] Adapted from Secondary Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

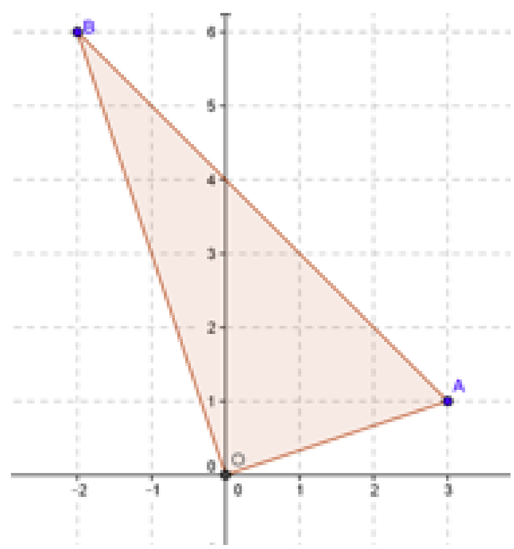
2. A quadrilateral has vertices $A = (0,0)$, $B = (1,3)$, $C = (0,4)$, and $D = (-1,1)$. Is $ABCD$ a parallelogram? How do you know?



3. A quadrilateral has vertices $A = (0,0)$, $B = (2,4)$, $C = (0,5)$, and $D = (-2,1)$. Is $ABCD$ a rectangle? How do you know?



4. Given the points $O\ (0,0)$, $A\ (3,1)$, and $B\ (-2,6)$, prove OA is perpendicular to OB.



(From Unit 3, Lesson 6)[3]

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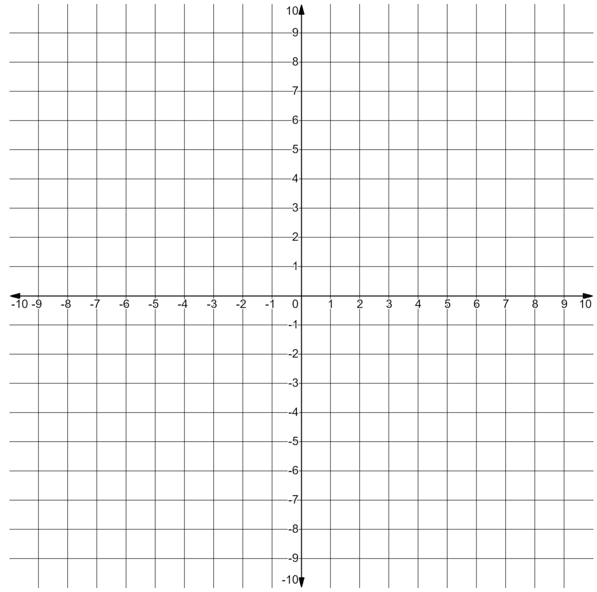
5. Han thinks that the points $(4,2)$ and $(-1,4)$ form a line perpendicular to a line with slope 4. Do you agree? Why or why not?[4]

(From Unit 3, Lesson 6)



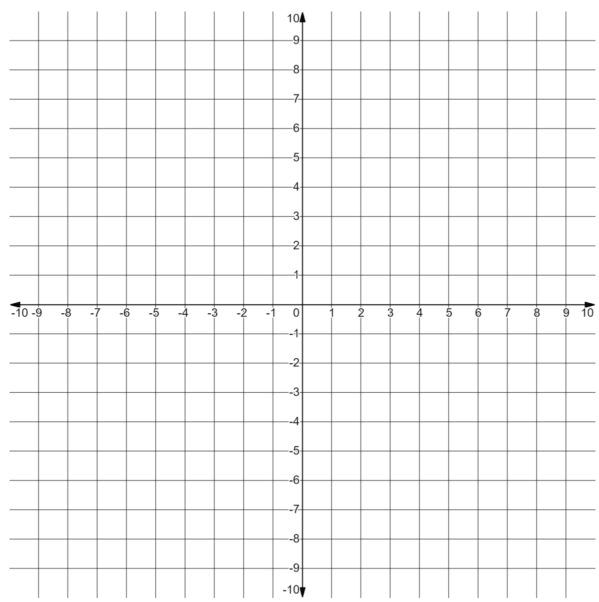
6. Write the equation of the line passing through $(-3,4)$ and perpendicular to $-2x+7y=-3$.

(From Unit 3, Lesson 6)[5]

7. Are the pairs of lines parallel, perpendicular or neither? Explain your reasoning.[6]

   a. $3x+2y=74$ and $9x-6y=15$

   b. $4x-9y=8$ and $18x+8y=7$



(From Unit 3, Lessons 5 and 6)

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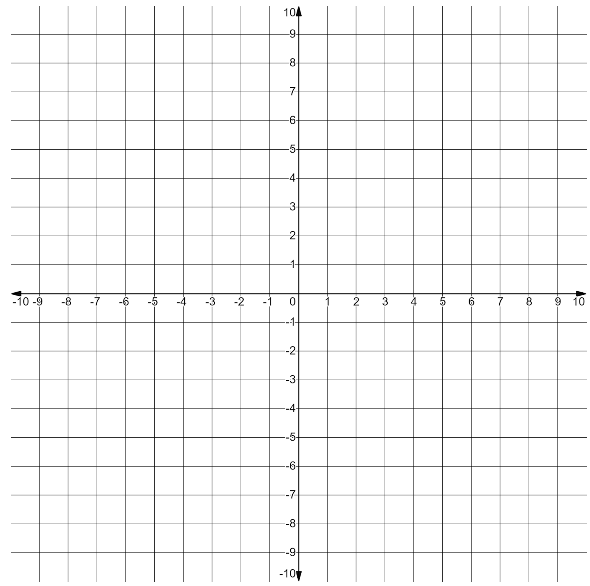
[5] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).
[6] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education (see above).

8. Write an equation of the line that passes through $(2,4)$ and has a slope of -3.



(From Unit 3, Lesson 4)

9. Consider the equation $2.5x + 5y = 20$. For each question, explain or show your reasoning.

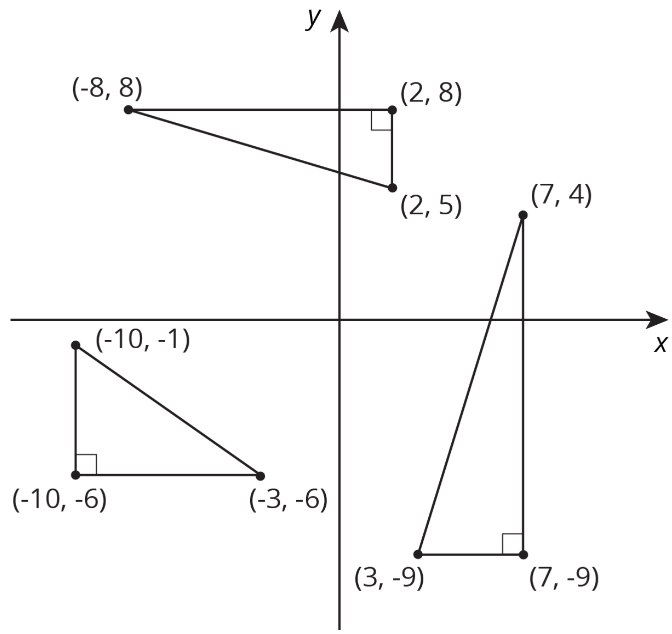
   a. If we graph the equation, what is the slope of the graph?

   b. Where does the graph intersect the y-axis?

   c. Where does it intersect the x-axis?

(From Unit 3, Lesson 3)

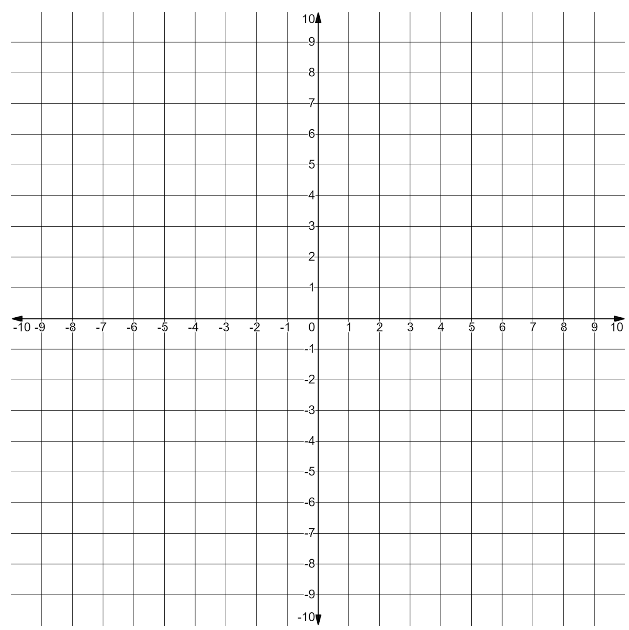
10. The right triangles are drawn in the coordinate plane, and the coordinates of their vertices are labeled. For each right triangle, label each leg with its length.[7]



(Addressing NC.8.G.8)

11. Find the distance between each pair of points. If you get stuck, try plotting the points.[8]

a. $M = (0, -11)$ and $P = (0, 2)$



b. $A = (0,0)$ and $B = (-3, -4)$

c. $C = (8, 0)$ and $D = (0, -6)$

(Addressing NC.8.G.8)

[7] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

[8] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).

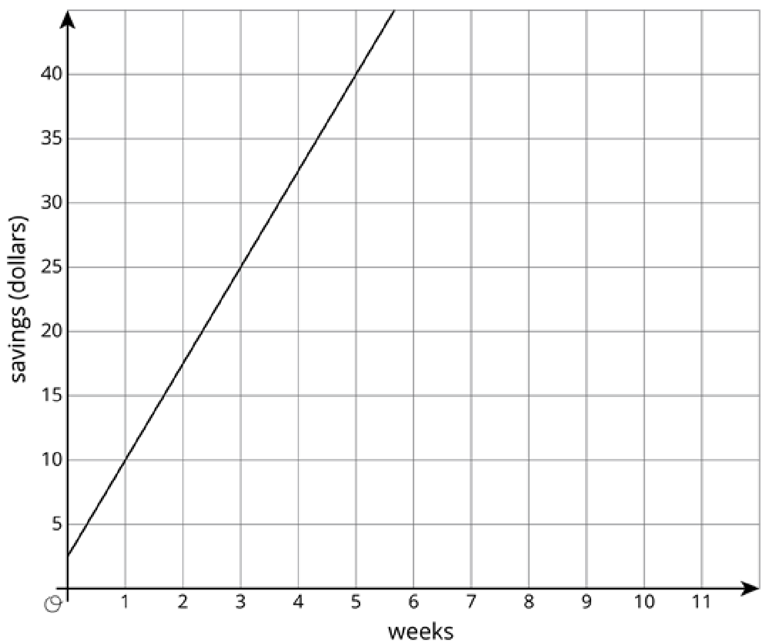
Lesson 8: Writing and Graphing Systems of Linear Equations

Learning Targets

- I can use tables and graphs to solve systems of equations.
- I can explain what we mean by "the solution to a system of linear equations" and can explain how the solution is represented graphically.
- I can explain what we mean when we refer to two equations as a system of equations.

Bridge

Andre and Noah started tracking their savings at the same time. Andre started with \$15 and deposits \$5 per week. Noah started with \$2.50 and deposits \$7.50 per week. The graph of Noah's savings is given and his equation is $y = 7.5x + 2.5$, where x represents the number of weeks and represents his savings.[1]



Write the equation for Andre's savings and graph it alongside Noah's.

What does the intersection point mean in this situation?

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Warm-up: A Possible Mix?

Diego bought some raisins and walnuts to make trail mix. Raisins cost $4 per pound, and walnuts cost $8 per pound. Diego spent $15 on both ingredients.

Decide if each pair of values could be a combination of raisins and walnuts that Diego bought.

1. 1.5 pounds of raisins and 1 pound of walnuts

2. 1 pound of raisins and 1.5 pounds of walnuts

3. 2.25 pounds of raisins and 0.75 pounds of walnuts

4. 3.5 pounds of raisins and 1 pound of walnuts

Activity 1: Trail Mix

1. Here is a situation you saw earlier: Diego bought some raisins and walnuts to make trail mix. Raisins cost $4 per pound and walnuts cost $8 per pound. Diego spent $15 on both ingredients.

    a. Write an equation to represent this constraint. Let x be the pounds of raisins and y be the pounds of walnuts.

    b. Use graphing technology to graph the equation.

    c. Complete the table with the amount of one ingredient Diego could have bought given the other. Be prepared to explain or show your reasoning.

Raisins (pounds)	Walnuts (pounds)
0	
0.25	
	1.375
	1.25
1.75	
3	

2. Here is a new piece of information: Diego bought a total of 2 pounds of raisins and walnuts combined.

    a. Write an equation to represent this new constraint. Let x be the pounds of raisins and y be the pounds of walnuts.

    b. Use graphing technology to graph the equation.

    c. Complete the table with the amount of one ingredient Diego could have bought given the other. Be prepared to explain or show your reasoning.

Raisins (pounds)	**Walnuts (pounds)**
0	
0.25	
	1.375
	1.25
1.75	
3	

3. Diego spent $15 and bought exactly 2 pounds of raisins and walnuts. How many pounds of each did he buy? Explain or show how you know.

Activity 2: Meeting Constraints

Here are two situations that each relate two quantities and involve two constraints. For each situation, find the pair of values that meet both constraints and explain or show your reasoning.

1. A dining hall had a total of 25 tables—some long rectangular tables and some round ones. Long tables can seat eight people. Round tables can seat six people. On a busy evening, all 190 seats at the tables are occupied.

   How many long tables, x, and how many round tables, y, are there?

2. A family bought a total of 16 adult and child tickets to a magic show. Adult tickets are \$10.50 each and child tickets are \$7.50 each. The family paid a total of \$141.

   How many adult tickets, a, and child tickets, c, did they buy?

Are You Ready For More?

1. Make up equations for two lines that intersect at $(4, 1)$.

2. Make up equations for three lines whose intersection points form a triangle with vertices at $(-4, 0)$, $(2, 9)$, and $(6, 5)$.

Lesson Debrief

Lesson 8 Summary and Glossary

A costume designer needs some silver and gold thread for the costumes for a school play. She needs a total of 240 yards. At a store that sells thread by the yard, silver thread costs $\$0.04$ a yard and gold thread costs $\$0.07$ a yard. The designer has $\$15$ to spend on the thread.

How many of each color should she get if she is buying exactly what is needed and spending all of her budget?

This situation involves two quantities and two constraints—length and cost. Answering the question means finding a pair of values that meets both constraints simultaneously. To do so, we can write two equations and graph them on the same coordinate plane.

Let x represents yards of silver thread and y yards of gold thread.

- The length constraint: $x + y = 240$
- The cost constraint: $0.04x + 0.07y = 15$

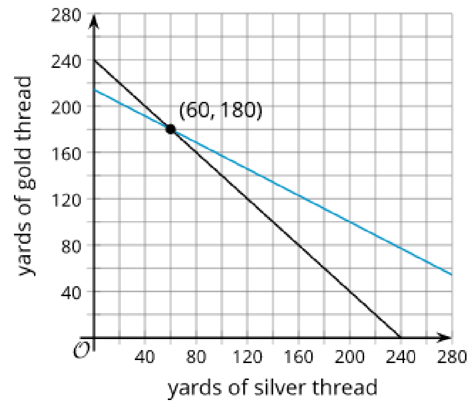
Every point on the graph of $x + y = 240$ is a pair of values that meets the length constraint.

Every point on the graph of $0.04x + 0.07y = 15$ is a pair of values that meets the cost constraint.

The point where the two graphs intersect gives the pair of values that meets *both* constraints.

That point is $(60, 180)$, which represents 60 yards of silver thread and 180 yards of gold thread.

If we substitute 60 for x and 180 for y in each equation, we find that these values make the equation true. $(60, 180)$ is a solution to both equations simultaneously.



$$\begin{aligned} x + y &= 240 \\ 60 + 180 &= 240 \\ 240 &= 240 \end{aligned}$$

$$\begin{aligned} 0.04x + 0.07y &= 15 \\ 0.04(60) + 0.07(180) &= 15 \\ 2.40 + 12.60 &= 15 \\ 15 &= 15 \end{aligned}$$

The equations we used to solve this problem are known as a **system of equations.**

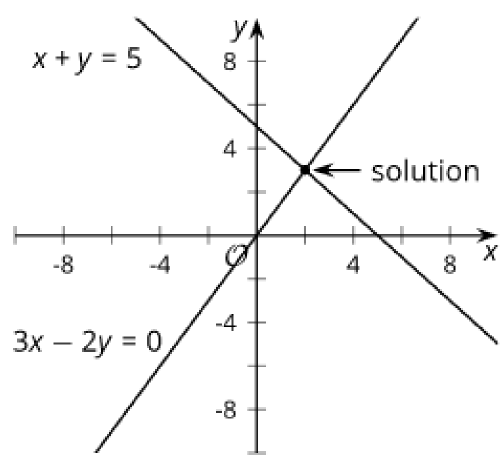
> **System of equations**: Two or more equations that represent the constraints in the same situation form a system of equations.

A curly bracket is often used to indicate a $\begin{cases} x + y = 240 \\ 0.04x + 0.07y = 15 \end{cases}$ system:

We say that this system of equations has the **solution** $(60, 180)$.

> **Solution to a system of equations**: A pair of values that makes all of the equations in the system true.

Graphing the equations is one way to find the solution to a system of equations. On the graph shown, the solution is the point where the graphs intersect.



Unit 3 Lesson 8 Practice Problems

1. The knitting club sold 40 scarves and hats at a winter festival and made $\$700$ from the sales. They charged $\$18$ for each scarf and $\$14$ for each hat.

   If s represents the number of scarves sold and h represents the number of hats sold, which system of equations represents the constraints in this situation?

   a. $\begin{cases} 40s + h = 700 \\ 18s + 14h = 700 \end{cases}$

   b. $\begin{cases} 18s + 14h = 40 \\ s + h = 700 \end{cases}$

   c. $\begin{cases} s + h = 40 \\ 18s + 14h = 700 \end{cases}$

   d. $\begin{cases} 40(s + h) = 700 \\ 18s = 14h \end{cases}$

2. Here are two equations:

   Equation 1: $6x + 4y = 34$

   Equation 2: $5x - 2y = 15$

   a. Decide whether each (x, y) pair is a solution to one equation, both equations, or neither of the equations.

      i. $(3, 4)$

      ii. $(4, 2.5)$

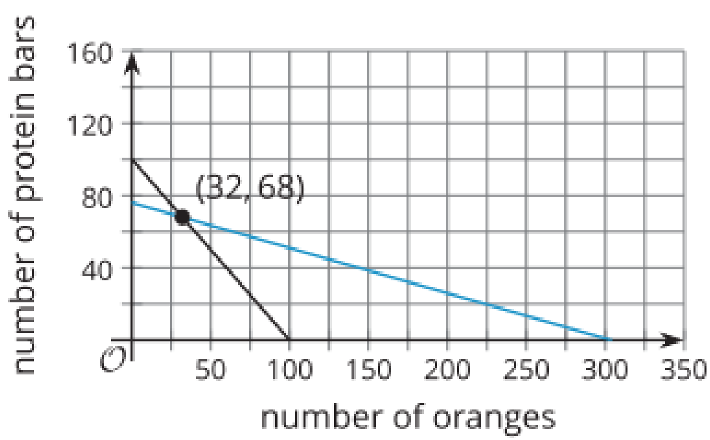
      iii. $(5, 5)$

      iv. $(3, 2)$

   b. Is it possible to have more than one (x, y) pair that is a solution to both equations? Explain or show your reasoning.

3. Explain or show that the point $(5, -4)$ is a solution to this system of equations: $\begin{cases} 3x - 2y = 23 \\ 2x + y = 6 \end{cases}$

4. A club is selling snacks at a track meet. Oranges cost $1 each and protein bars cost $4 each. They sell a total of 100 items, and collect $304.



a. Write two equations that represent this situation.

b. What does the solution $(32, 68)$ represent?

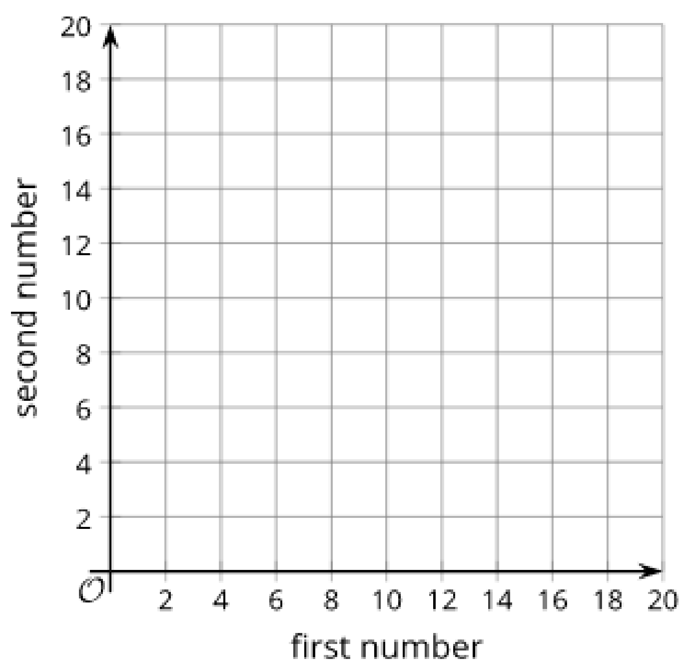
5. Diego is thinking of two positive numbers. He says, “If we triple the first number and double the second number, the sum is 34.”

   a. Write an equation that represents this clue. Then, find two possible pairs of numbers Diego could be thinking of.

   b. Diego then says, “If we take half of the first number and double the second, the sum is 14.”

      Write an equation that could represent this description.

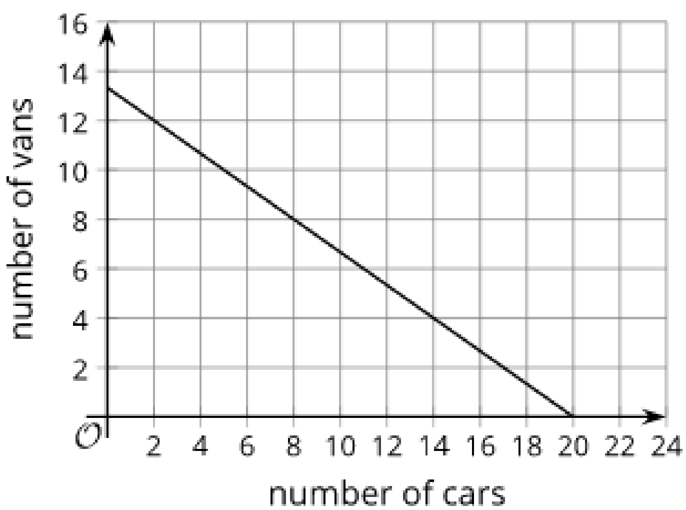
   c. What are Diego’s two numbers? Explain or show how you know. A coordinate plane is given here, in case helpful.



6. At a poster shop, Han paid $16.80 for two large posters and three small posters of his favorite band. Kiran paid $14.15 for one large poster and four small posters of his favorite TV shows. Posters of the same size have the same price.

   Find the price of a large poster, ℓ, and the price of a small poster, s.

7. Volunteer drivers are needed to bring 80 students to the championship baseball game. Drivers either have cars, which can seat four students, or vans, which can seat six students. The equation $4c + 6v = 80$ describes the relationship between the number of cars, c, and number of vans, v, that can transport exactly 80 students.

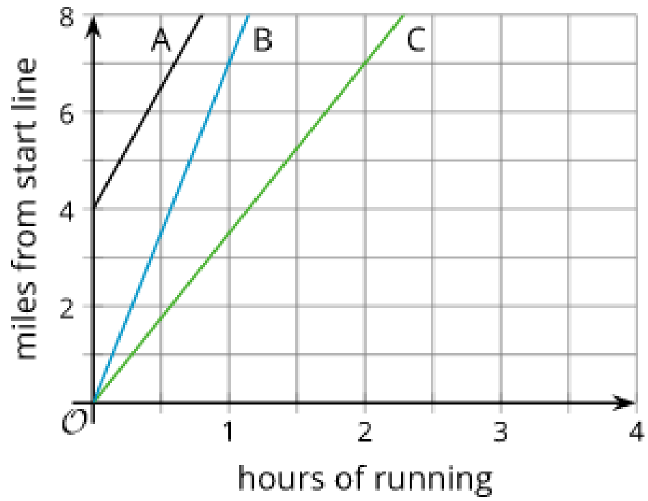


   Explain how you know that this graph represents this equation.

(From Unit 3, Lesson 2)

8. Three siblings are participating in a family-friendly running event.

    - The oldest sibling begins at the start line of the race and runs 7 miles per hour the entire time.
    - The middle sibling begins at the start line and walks at 3.5 miles per hour throughout the race.
    - The youngest sibling joins the race 4 miles from the start line and runs 5 miles per hour the rest of the way.



Match each graph to the sibling whose running is represented by the graph below.

Graph	Sibling
Graph A Graph B Graph C	1. Oldest Sibling 2. Middle Sibling 3. Youngest Sibling

(From Unit 3, Lesson 6)

9. What is the x-intercept of the graph of $y = 3 - 5x$?

    a. $(\frac{3}{5}, 0)$

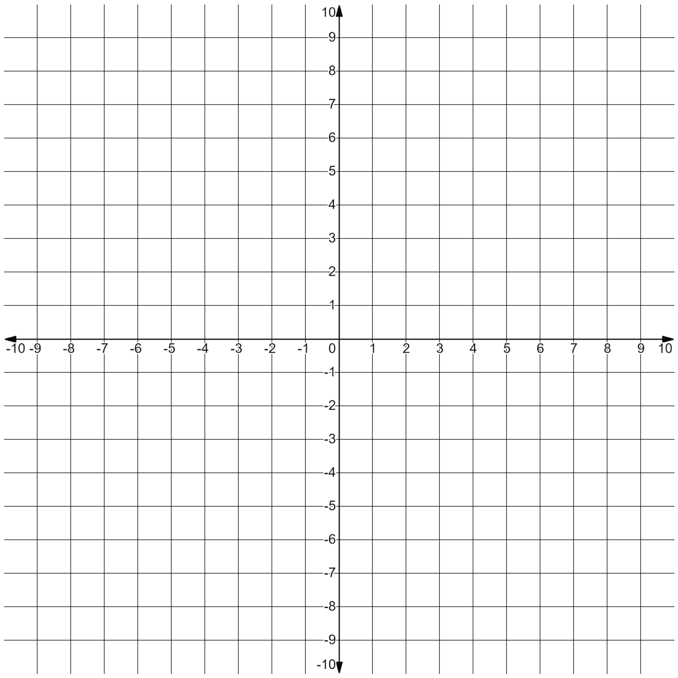
    b. $(-5, 0)$

    c. $(0, 3)$

    d. $(0, \frac{5}{3})$

(From Unit 3, Lesson 6)

10. A triangle has vertices $C(-3,-2)$, $A(2,5)$, and $T(6,-1)$. Is CAT isosceles? How do you know?



(From Unit 3, Lesson 7)

11. Match each equation with the corresponding equation solved for a.

a. $a+2b=5$

b. $5a=2b$

c. $a+5=2b$

d. $5(a+2b)=0$

e. $5a+2b=0$

1. $a=\frac{2b}{5}$
2. $a=\frac{-2b}{5}$
3. $a=-2b$
4. $a=2b-5$
5. $a=5-2b$

(From Unit 2)

12. Andre does not understand why a solution to the equation $3 - x = 4$ must also be a solution to the equation $12 = 9 - 3x$.

    Write a convincing explanation as to why this is true.

    (From Unit 2)

13. Priya's cell phone plan costs $35 each month plus $15 for each gigabyte of data she uses. Han's plan costs $75 each month, plus $5 for each gigabyte of data. Is there a number of gigabytes of data for which the plans cost the same amount? How many gigabytes will they use to pay the same amount?

    (Addressing NC.8.EE.8)

Lesson 9: Solving Systems by Substitution

Learning Targets

- I can solve systems of equations by substituting a variable with a number or an expression.
- I know more than one way to perform substitution and can decide which way or what to substitute based on how the given equations are written.

Bridge

Find the value of y when $x = 5$.

a. $y = 3x - 4$

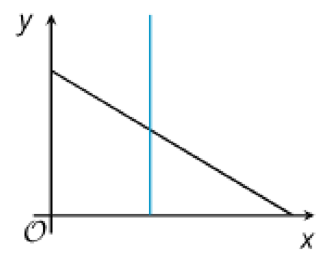
b. $y = \frac{2}{5}x + 4$

c. $y = 2x + 3 + (3x - 1)$

d. $y = 4x - (x + 1)$

Warm-up: Is It a Match?

Here are graphs of two equations in a system.



Choose two of these systems and determine if each of them could be represented by the graphs. Be prepared to explain how you know.

a. $\begin{cases} x + 2y = 8 \\ x = -5 \end{cases}$

b. $\begin{cases} y = -7x + 13 \\ y = -1 \end{cases}$

c. $\begin{cases} 3x = 8 \\ 3x + y = 15 \end{cases}$

d. $\begin{cases} y = 2x - 7 \\ 4 + y = 12 \end{cases}$

Activity 1: Four Systems

Here are four systems of equations you saw earlier. Solve each system. Then, check your solutions by substituting them into the original equations to see if the equations are true.

a. $\begin{cases} x + 2y = 8 \\ x = -5 \end{cases}$

b. $\begin{cases} y = -7x + 13 \\ y = -1 \end{cases}$

c. $\begin{cases} 3x = 8 \\ 3x + y = 15 \end{cases}$

d. $\begin{cases} y = 2x - 7 \\ 4 + y = 12 \end{cases}$

Activity 2: What about Now?

Solve each system without graphing.

a. $\begin{cases} 5x - 2y = 26 \\ y + 4 = x \end{cases}$

b. $\begin{cases} 2m - 2p = -6 \\ p = 2m + 10 \end{cases}$

c. $\begin{cases} 2d = 8f \\ 18 - 4f = 2d \end{cases}$

d. $\begin{cases} w + \frac{1}{7}z = 4 \\ z = 3w - 2 \end{cases}$

Are You Ready For More?

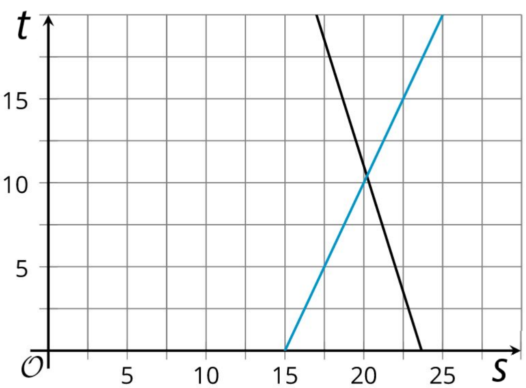
Solve this system with four equations.

$$\begin{cases} 3x + 2y - z + 5w = 20 \\ y = 2z - 3w \\ z = w + 1 \\ 2w = 8 \end{cases}$$

Lesson Debrief

Lesson 9 Summary and Glossary

The solution to a system can usually be found by graphing, but graphing may not always be the most precise or the most efficient way to solve a system.



Here is a system of equations: $\begin{cases} 3s + t = 71 \\ 2s - t = 30 \end{cases}$

The graphs of the equations show an intersection point at approximately 20 for s and approximately 10 for t.

Without technology, however, it is not easy to tell what the exact values are.

Instead of solving by graphing, we can solve the system algebraically. Here is one way.

If we subtract $3s$ from each side of the first equation, $3s + t = 71$, we get an equivalent equation: $t = 71 - 3s$. Rewriting the original equation this way allows us to isolate the variable t.

Because t is equal to $71 - 3s$, we can substitute the expression $71 - 3s$ in the place of t in the second equation. Doing this gives us an equation with only one variable, s, and makes it possible to find s.

$$\begin{aligned} 2s - t &= 30 && \text{original equation} \\ 2s - (71 - 3s) &= 30 && \text{substitute } 71 - 3s \text{ for } t \\ 2s - 71 + 3s &= 30 && \text{apply distributive property} \\ 5s - 71 &= 30 && \text{combine like terms} \\ 5s &= 101 && \text{add 71 to both sides} \\ s &= \tfrac{101}{5} && \text{divide both sides by 5} \\ s &= 20.2 \end{aligned}$$

Now that we know the value of s, we can find the value of t by substituting 20.2 for s in either of the original equations and solving the equation.

$$\begin{aligned} 3(20.02) + t &= 71 \\ 60.6 + t &= 71 \\ t &= 71 - 60.6 \\ t &= 10.4 \end{aligned}$$

$$\begin{aligned} 2(20.2) - t &= 30 \\ 40.4 - t &= 30 \\ -t &= 30 - 40.4 \\ -t &= -10.4 \\ t &= \tfrac{-10.4}{-1} \\ t &= 10.4 \end{aligned}$$

The solution to the system is the pair $s = 20.2$ and $t = 10.4$, or the point $(20.2, 10.4)$ on the graph.

This method of solving a system of equations is called solving by *substitution*, because we substituted an expression for t into the second equation.

Unit 3 Lesson 9 Practice Problems

1. Identify a solution to this system of equations: $\begin{cases} -4x+3y=23 \\ x-y=-7 \end{cases}$

   a. $(-5,2)$

   b. $(-2,5)$

   c. $(-3,4)$

   d. $(4,-3)$

2. Lin is solving this system of equations: $\begin{cases} 6x-5y=34 \\ 3x+2y=8 \end{cases}$

   She starts by rearranging the second equation to isolate the y variable: $y=4-1.5x$. She then substituted the expression $4-1.5x$ for y in the first equation, as shown:

$$\begin{aligned} 6x-5(4-1.5x)&=34 \\ 6x-20-7.5x&=34 \\ -1.5x&=54 \\ x&=-36 \end{aligned}$$

$$\begin{aligned} y&=4-1.5x \\ y&=4-1.5\cdot(-36) \\ y&=58 \end{aligned}$$

   a. Check to see if Lin's solution of $(-36,58)$ makes both equations in the system true.

   b. If your answer to the previous question is "no," find and explain her mistake. If your answer is "yes," graph the equations to verify the solution of the system.

3. Solve each system of equations.

a. $\begin{cases} 2x - 4y = 20 \\ x = 4 \end{cases}$

b. $\begin{cases} y = 6x + 11 \\ 2x - 3y = 7 \end{cases}$

4. Tyler and Han are trying to solve this system by substitution: $\begin{cases} x + 3y = -5 \\ 9x + 3y = 3 \end{cases}$

Tyler's first step is to isolate x in the first equation to get $x = -5 - 3y$. Han's first step is to isolate $3y$ in the first equation to get $3y = -5 - x$.

Show that both first steps can be used to solve the system and will yield the same solution.

Elena wanted to find the slope and y-intercept of the graph of $25x - 20y = 100$. She decided to put the equation in slope-intercept form first. Here is her work:

$$25x - 20y = 100$$
$$20y = 100 - 25x$$
$$y = 5 - \frac{5}{4}x$$

She concluded that the slope is $-\frac{5}{4}$ and the y-intercept is $(0, 5)$.

a. What was Elena's mistake?

b. What are the slope and y-intercept of the line? Explain or show your reasoning.

(From Unit 3, Lesson 3)

6. Find the x- and y-intercepts of the graph of each equation.

a. $y = 10 - 2x$

b. $4y + 9x = 18$

c. $6x - 2y = 44$

d. $2x = 4 + 12y$

(From Unit 3, Lesson 3)

7. Andre is buying snacks for the track and field team. He buys a pounds of apricots for $\$6$ per pound and b pounds of dried bananas for $\$4$ per pound. He buys a total of 5 pounds of apricots and dried bananas, and spends a total of $\$24.50$.

Which system of equations represents the constraints in this situation?

a. $\begin{cases} 6a + 4b = 5 \\ a + b = 24.50 \end{cases}$

b. $\begin{cases} 6a + 4b = 24.50 \\ a + b = 5 \end{cases}$

c. $\begin{cases} 6a = 4b \\ 5(a + b) = 24.50 \end{cases}$

d. $\begin{cases} 6a + b = 4 \\ 5a + b = 24.50 \end{cases}$

(From Unit 3, Lesson 8)

8. Here are two equations:

   Equation 1: $y = 3x + 8$

   Equation 2: $2x - y = -6$

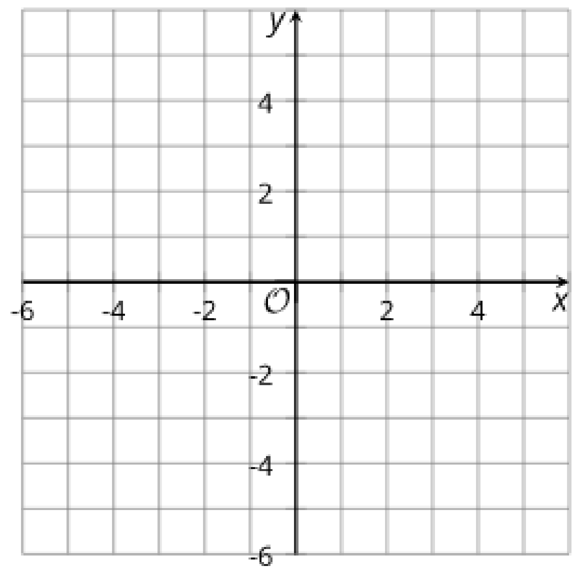
   Without using graphing technology:

   a. Find a point that is a solution to equation 1 but not a solution to equation 2.

   b. Find a point that is a solution to equation 2 but not a solution to equation 1.

   c. Graph the two equations.

   d. Find a point that is a solution to both equations.



(From Unit 3, Lesson 8)

9. Kiran buys supplies for the school's greenhouse. He buys f bags of fertilizer and p packages of soil. He pays $\$5$ for each bag of fertilizer and $\$2$ for each package of soil, and spends a total of $\$90$. The equation $5f + 2p = 90$ describes this relationship.

   If Kiran solves the equation for p, which equation would result?

   a. $2p = 90 - 5f$

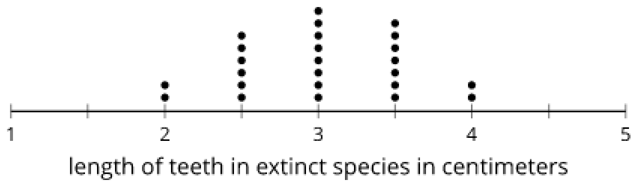
   b. $p = \frac{5f - 90}{2}$

   c. $p = 45 - 2.5f$

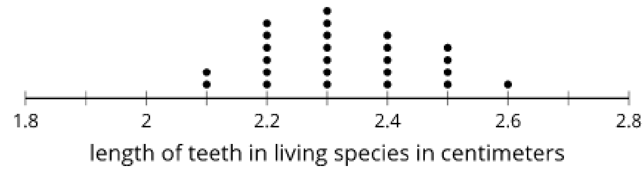
   d. $p = \frac{85f}{2}$

(From Unit 2)

10. The dot plots show the distribution of the length, in centimeters, of 25 shark teeth for an extinct species of shark and the length, in centimeters, of 25 shark teeth for a closely related shark species that is still living.



mean: 3.02 cm
standard deviation: 0.55 cm



mean: 2.32 cm
standard deviation: 0.13 cm

Compare the two dot plots using the shape of the distribution, measures of center, and measures of variability. Use the situation described in the problem in your explanation.

(From Unit 1)

11. For each equation, find the value of y when $x = -3$.

a. $y = 4x$

b. $y = -4x + 2$

c. $y = -(4x + 2)$

d. $y = 4x - 2$

(Addressing NC.8.EE.7)

Lesson 10: Solving Systems by Elimination (Part One)

Learning Target

- I can solve systems of equations by adding or subtracting them to eliminate a variable.

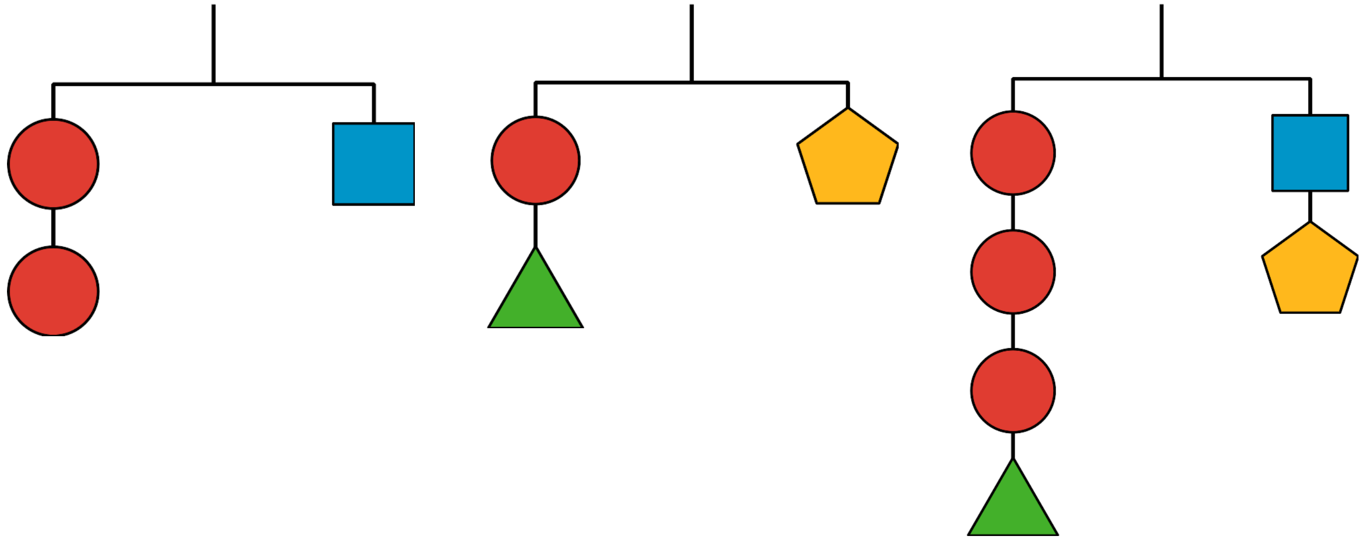
Bridge

Rewrite each expression by combining like terms.

1. $11s - 2s$
2. $-4x + 6y - (7x + 2y)$
3. $8x - 3y + (3y - 5x)$
4. $5x + 4y - (5x + 7y)$

Warm-up: Hanger Diagrams

What do you notice? What do you wonder?



Activity 1: Adding Equations

Diego is solving this system of equations:

$$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases}$$

Here is his work:

$$\begin{array}{rr} & 4x + 3y = 10 \\ + & (-4x + 5y = 6) \\ \hline & 0 + 8y = 16 \\ & y = 2 \end{array}$$

$$\begin{aligned} 4x + 3(2) &= 10 \\ 4x + 6 &= 10 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

1. Make sense of Diego's work and discuss with a partner:

   a. What did Diego do to solve the system?

   b. Is the pair of x and y values that Diego found actually a solution to the system? How do you know?

2. Does Diego's method work for solving these systems? Be prepared to explain or show your reasoning.

   a. $\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$

   b. $\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$

Activity 2: A Bunch of Systems

Solve each system of equations without graphing and show your reasoning. Then, check your solutions.

1. $\begin{cases} 2x + 3y = 7 \\ -2x + 4y = 14 \end{cases}$

2. $\begin{cases} 2x + 3y = 7 \\ 3x - 3y = 3 \end{cases}$

3. $\begin{cases} 2x + 3y = 5 \\ 2x + 4y = 9 \end{cases}$

4. $\begin{cases} 2x + 3y = 16 \\ 6x - 5y = 20 \end{cases}$

Are You Ready For More?

This system has three equations:

$$\begin{cases} 3x + 2y - z = 7 \\ -3x + y + 2z = -14 \\ 3x + y - z = 10 \end{cases}$$

1. Add the first two equations to get a new equation.
2. Add the second two equations to get a new equation.
3. Solve the system of your two new equations.
4. What is the solution to the original system of equations?

Lesson Debrief

Lesson 10 Summary and Glossary

Another way to solve systems of equations algebraically is by *elimination*. Just like in substitution, the idea is to eliminate one variable so that we can solve for the other. This can be done by adding or subtracting equations in the system. Let's look at an example.

$$\begin{cases} 5x + 7y = 64 \\ 0.5x - 7y = -9 \end{cases}$$

Notice that one equation has $7y$ and the other has $-7y$.

If we add the second equation to the first, the $7y$ and $-7y$ add up to 0, which eliminates the y-variable, allowing us to solve for x.

$$\begin{array}{rr} & 5x + 7y = 64 \\ + & (0.5x - 7y = -9) \\ \hline & 5.5x + 0 = 55 \\ & 5.5x = 55 \\ & x = 10 \end{array}$$

Now that we know $x = 10$, we can substitute 10 for x in either of the equations and find y:

$$\begin{aligned} 5x + 7y &= 64 \\ 5(10) + 7y &= 64 \\ 50 + 7y &= 64 \\ 7y &= 14 \\ y &= 2 \end{aligned}$$

$$\begin{aligned} 0.5x - 7y &= -9 \\ 0.5(10) - 7y &= -9 \\ 5 - 7y &= -9 \\ -7y &= -14 \\ y &= 2 \end{aligned}$$

In this system, the coefficient of y in the first equation happens to be the opposite of the coefficient of y in the second equation. The sum of the terms with y-variables is 0.

What if the equations don't have opposite coefficients for the same variable, like in the following system?

$$\begin{cases} 8r + 4s = 12 \\ 8r + s = -3 \end{cases}$$

Notice that both equations have $8r$ and if we subtract the second equation from the first, the variable r will be eliminated because $8r - 8r$ is 0.

$$\begin{array}{rr} & 8r + 4s = 12 \\ - & (8r + s = -3) \\ \hline & 0 + 3s = 15 \\ & 3s = 15 \\ & s = 5 \end{array}$$

Substituting 5 for s in one of the equations gives us r:

$$\begin{aligned} 8r + 4s &= 12 \\ 8r + 4(5) &= 12 \\ 8r + 20 &= 12 \\ 8r &= -8 \\ r &= -1 \end{aligned}$$

Unit 3 Lesson 10 Practice Problems

1. Which equation is the result of adding these two equations? $\begin{cases} -2x + 4y = 17 \\ 3x - 10y = -3 \end{cases}$

    a. $-5x - 6y = 14$

    b. $-x - 6y = 14$

    c. $x - 6y = 14$

    d. $5x + 14y = 20$

2. Which equation is the result of subtracting the second equation from the first? $\begin{cases} 4x - 6y = 13 \\ -5x + 2y = 5 \end{cases}$

    a. $-9x - 4y = 8$

    b. $-x + 4y = 8$

    c. $x - 4y = 8$

    d. $9x - 8y = 8$

3. Solve this system of equations without graphing: $\begin{cases} 5x + 2y = 29 \\ 5x - 2y = 41 \end{cases}$

4. Here is a system of linear equations: $\begin{cases} 6x + 21y = 103 \\ -6x + 23y = 51 \end{cases}$

   Would you rather use subtraction or addition to solve the system? Explain your reasoning.

5. Elena and Kiran are playing a board game. After one round, Elena says, "You earned so many more points than I did. If you earned 5 more points, your score would be twice mine!"

   Kiran says, "Oh, I don't think I did that much better. I only scored 9 points higher than you did."

   a. Write a system of equations to represent each student's comment. Be sure to specify what your variables represent.

   b. If both students were correct, how many points did each student score? Show your reasoning.

(From Unit 3, Lesson 9)

6. Solve each system of equations by substitution.

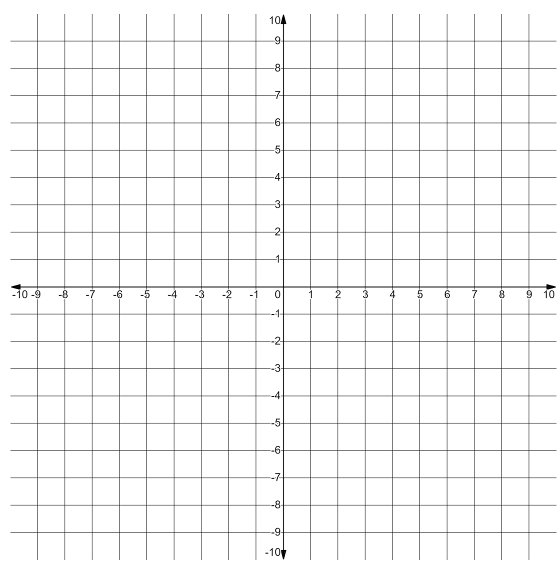
a. $\begin{cases} 2x + 3y = 4 \\ 2x = 7y + 24 \end{cases}$

b. $\begin{cases} 5x + 3y = 23 \\ 3y = 15x - 21 \end{cases}$

(From Unit 3, Lesson 9)

7. A triangle has vertices $A(-6, -3)$, $N(6, 5)$, and $T(2, -2)$.

Is ANT isosceles? How do you know?



(From Unit 3, Lesson 7)

8. Match each equation with the slope m and y-intercept of its graph.

Slope and y-intercept	**Equation**
a. $m = -6, y - int = (0, 12)$	1. $5x - 6y = 30$
b. $m = -6, y - int = (0, 5)$	2. $y = 5 - 6x$
c. $m = -\frac{5}{6}, y - int = (0, 1)$	3. $y = \frac{5}{6}x + 1$
d. $m = \frac{5}{6}, y - int = (0, 1)$	4. $5x - 6y = 6$
e. $m = \frac{5}{6}, y - int = (0, -1)$	5. $5x + 6y = 6$
f. $m = \frac{5}{6}, y - int = (0, -5)$	6. $6x + y = 12$

(From Unit 3, Lesson 3)

9. Andre sells f full boxes and h half-boxes of fruit to raise money for a band trip. He earns \$5 for each full box and \$2 for each half-box of fruit he sells and earns a total of \$100 toward the cost of his band trip. The equation $5f + 2h = 100$ describes this relationship.

   Solve the equation for f.

(From Unit 2)

10. The volume of a cylinder is represented by the formula $V = \pi r^2 h$.

Find each missing height and show your reasoning.

Volume (cubic inches)	**Radius (inches)**	**Height (inches)**	**Show your reasoning.**
96π	4		
31.25π	2.5		
V	r		

(From Unit 2)

11. Select **all** the expressions that are equivalent to $5x + 30x - 15x$.

a. $5(x + 6x - 3x)$

b. $(5 + 30 - 15) \cdot x$

c. $x(5 + 30x - 15x)$

d. $5x(1 + 6 - 3)$

e. $5(x + 30x - 15x)$

(Addressing NC.6.EE.4)[1]

Lesson 11: Solving Systems by Elimination (Part Two)

Learning Targets

- I understand that multiplying each side of an equation by a factor creates an equivalent equation whose graph and solutions are the same as that of the original equation.
- I can solve systems of equations by multiplying each side of one or both equations by a factor, then adding or subtracting the equations to eliminate a variable.

Warm-up: Multiplying Equations by a Factor

Consider two equations in a system:

$$\begin{cases} 4x + y = 1 & \text{Equation A} \\ x + 2y = 9 & \text{Equation B} \end{cases}$$

1. Use graphing technology to graph the equations. Then, identify the coordinates of the solution.
2. Write equations that are equivalent to equation A by multiplying both sides of it by the same number, for example, 2 or -5. Let's call the resulting equations A1 and A2. Record your equations here:
    a. Equation A1:

    b. Equation A2:

3. Using technology, graph the equations you generated. Make a couple of observations about the graphs.

Activity 1: Writing a New System to Solve a Given System

Here is a system you solved by graphing earlier.

$$\begin{cases} 4x + y = 1 & \text{equation A} \\ x + 2y = 9 & \text{equation B} \end{cases}$$

To start solving the system, Elena wrote:

$$\begin{array}{c} 4x + y = 1 \\ x + 2y = 9 \end{array} \longrightarrow \begin{array}{c} 4x + y = 1 \\ 4x + 8y = 36 \end{array}$$

And then she wrote:

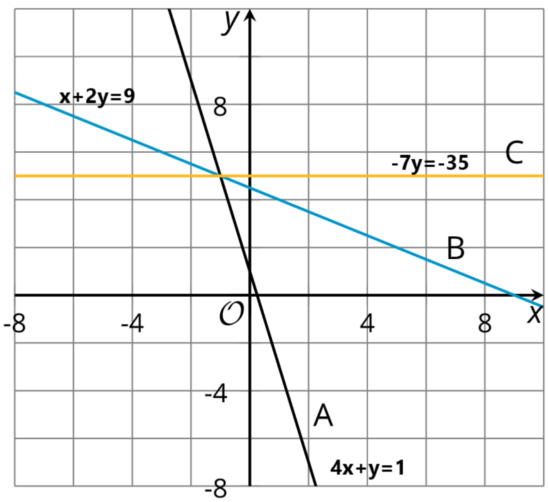
$$\begin{array}{c} 4x + y = 1 \\ 4x + 8y = 36 \end{array} \longrightarrow \begin{array}{r} 4x + y = 1 \\ -\quad (4x + 8y = 36) \\ \hline -7y = -35 \end{array}$$

1. What were Elena's first two moves? What might be possible reasons for those moves?

2. Complete the solving process algebraically. Show that the solution is indeed $x = -1, y = 5$.

3. Elena wanted to check her solution from a different perspective. She graphed the original equations of the system $4x + y = 1$, $x + 2y = 9$, as well as the equation she got by combining the two equations: $-7y = -35$.

   What do you notice? What do you wonder?



First draft

"It is not surprising that $(-1, 5)$ is the solution to Elena's new system, just because her method eliminated x by first multiplying to rewrite one equation.Then she subtracted something the same from both sides of an equation with that solution, so her method didn't really change anything, even the solution."

Activity 2: Classroom Supplies

A teacher purchased 20 calculators and 10 measuring tapes for her class and paid $495. Later, she realized that she didn't order enough supplies. She placed another order of eight of the same calculators and one more of the same measuring tape and paid $178.50.

1. Write a system of equations representing the constraints in the situation. Let c represent the price of a calculator and m represent the price of a measuring tape.

2. Solve the system of equations without graphing. Explain or show your reasoning.

3. What does the solution to the system mean in this situation?

Activity 3: Build Some Equivalent Systems

Here is a system of equations:

$$\begin{cases} 6x + 5y = -7 \\ 2x - 10y = -14 \end{cases}$$

To solve this system, Diego wrote the following equivalent system:

$$\begin{array}{c} 6x + 5y = -7 \\ 2x - 10y = -14 \end{array} \longrightarrow \begin{array}{rr} & 12x + 10y = -14 \\ + & (2x - 10y = -14) \\ \hline & 14x = -28 \end{array}$$

1. Describe the moves that Diego made to create the equivalent system and eliminate the y-variable.

2. Write another equivalent system (different than Diego's) that will allow one variable to be eliminated and enable you to solve the original system. Be prepared to describe the moves you make to create the equivalent system.

3. Use your equivalent system to solve the original system. Then, check your solution by substituting the pair of values into the original system.

Lesson Debrief

Lesson 11 Summary and Glossary

We now have two algebraic strategies for solving systems of equations: substitution and elimination. In some systems, the equations may give us a clue as to which strategy to use. For example:

$$\begin{cases} y = 2x - 11 \\ 3x + 2y = 18 \end{cases}$$

In this system, y is already isolated in one equation. We can solve the system by substituting $2x-11$ for y in the second equation and finding x.

$$\begin{cases} 3x - y = -17 \\ -3x + 4y = 23 \end{cases}$$

This system is set up nicely for elimination because of the opposite coefficients of the x-variable. Adding the two equations eliminates x so we can solve for y.

In other systems, which strategy to use is less straightforward, either because no variables are isolated, or because no variables have equal or opposite coefficients. For example:

$$\begin{cases} 6x - 12y = 24 \quad \text{equation A} \\ -x + 3y = -5 \quad \text{equation B} \end{cases}$$

To solve this system by elimination, we first need to rewrite one or both equations so that one variable can be eliminated. To do that, we can multiply each side of an equation by the same factor. Remember that doing this doesn't change the equality of the two sides of the equation, so the x- and y-values that make the first equation true also make the new equation true.

There are different ways to eliminate a variable with this approach. For instance, we could:

Multiply equation B by 6 to get $-6x+18y=-30$. Adding the equations of the equivalent system will result in eliminating the x variable.

$$\begin{array}{r} 6x-12y=24 \\ -x+3y=-5 \end{array} \longrightarrow \begin{array}{r} 6x-12y=24 \\ -6x+18y=-30 \end{array}$$

Multiply equation B by 4 to get $-4x+12y=-20$. Adding the equations of the equivalent system will result in eliminating the y-variable.

$$\begin{array}{r} 6x-12y=24 \\ -x+3y=-5 \end{array} \longrightarrow \begin{array}{r} 6x-12y=24 \\ -4x+12y=-20 \end{array}$$

Multiply equation A by $\frac{1}{6}$ to get $x-2y=4$. Adding the equations of the equivalent system will result in eliminating the x-variable.

$$\begin{array}{r} 6x-12y=24 \\ -x+3y=-5 \end{array} \longrightarrow \begin{array}{r} x-2y=4 \\ -x+3y=-5 \end{array}$$

Each multiple of an original equation is equivalent to the original equation. So each new pair of equations is an equivalent system to the original system and has the same solution.

Let’s solve the original system using the first equivalent system we found earlier.

Adding the equations eliminates the x, leaving a new equation $6y=-6$ or $y=-1$.

$$\begin{array}{r} 6x-12y=24 \\ -x+3y=-5 \end{array} \longrightarrow \begin{array}{r} 6x-12y \\ +\quad(-6x+18y= \\ \hline 6y=- \\ y=- \end{array}$$

Substituting -1 for y in the second equation allows us to solve for x.

$$\begin{aligned} -x+3(-1)&=-5 \\ -x-3&=-5 \\ -x&=-2 \\ x&=2 \end{aligned}$$

Unit 3 Lesson 11 Practice Problems

1. Solve each system of equations.

a. $\begin{cases} 2x - 4y = 10 \\ x + 5y = 40 \end{cases}$

b. $\begin{cases} 3x - 5y = 4 \\ -2x + 6y = 18 \end{cases}$

2. Tyler is solving this system of equations: $\begin{cases} 4p + 2q = 62 \\ 8p - q = 59 \end{cases}$

He can think of two ways to eliminate a variable and solve the system:

- Multiply $4p + 2q = 62$ by 2, then subtract $8p - q = 59$ from the result.
- Multiply $8p - q = 59$ by 2, then add the result to $4p + 2q = 62$.

Do both strategies work for solving the system? Explain or show your reasoning.

3. Andre and Elena are solving this system of equations: $\begin{cases} y = 3x \\ y = 9x - 30 \end{cases}$

   Andre's first step is to write: $3x = 9x - 30$

   Elena's first step is to create a new system: $\begin{cases} 3y = 9x \\ y = 9x - 30 \end{cases}$

   Do you agree with either first step? Explain your reasoning.

4. Select **all** systems that are equivalent to this system: $\begin{cases} 6d + 4.5e = 16.5 \\ 5d + 0.5e = 4 \end{cases}$

   a. $\begin{cases} 6d + 4.5e = 16.5 \\ 45d + 4.5e = 4 \end{cases}$

   b. $\begin{cases} 30d + 22.5e = 82.5 \\ 5d + 0.5e = 4 \end{cases}$

   c. $\begin{cases} 30d + 22.5e = 82.5 \\ 30d + 3e = 24 \end{cases}$

   d. $\begin{cases} 6d + 4.5e = 16.5 \\ 6d + 0.6e = 4.8 \end{cases}$

   e. $\begin{cases} 12d + 9e = 33 \\ 10d + 0.5e = 8 \end{cases}$

   f. $\begin{cases} 6d + 4.5e = 16.5 \\ 11d + 5e = 20.5 \end{cases}$

5. Here is a system of equations with a solution: $\begin{cases} p + 8q = -8 \\ \frac{1}{2}p + 5q = -5 \end{cases}$

   a. Write a system of equations that is equivalent to this system. Describe what you did to the original system to get the new system.

   b. Explain how you know the new system has the same solution as the original system.

6. Here is a system of equations: $\begin{cases} -7x + 3y = -65 \\ -7x + 10y = -135 \end{cases}$

   Write an equation that results from subtracting the two equations.

   (From Unit 3, Lesson 10)

7. Here is a system of linear equations: $\begin{cases} 2x + 7y = 8 \\ y + 2x = 14 \end{cases}$

   Find at least one way to solve the system by substitution and show your reasoning. How many ways can you find? (Regardless of the substitution that you do, the solution should be the same.)

   (From Unit 3, Lesson 9)

8. A grocery store sells bananas for b dollars per pound and grapes for g dollars per pound. Priya buys 2.2 pounds of bananas and 3.6 pounds of grapes for \$9.35. Andre buys 1.6 pounds of bananas and 1.2 pounds of grapes for \$3.68.

   Write a system of equations to represent the constraints in this situation.

   (From Unit 3, Lesson 9)

9. Noah wants to mail a package to his friend and the cost of mailing it is \$5.00. Noah asked his mom for \$5.00 but instead she gave him some postcard stamps that are worth \$0.34 each and some first-class stamps that are worth \$0.49 each.

   a. Write an equation that relates the number of postcard stamps , p , the number of first-class stamps, f , and the cost of mailing the package.

   b. Solve the equation for f .

   c. Solve the equation for p .

   d. If Noah puts seven first-class stamps on the package, how many postcard stamps will he need?

   (From Unit 2)

Lesson 12: Systems of Linear Equations and Their Solutions

Learning Targets

- I can tell how many solutions a system has by graphing the equations or by analyzing the parts of the equations and considering how they affect the features of the graphs.
- I know the possibilities for the number of solutions a system of equations could have.

Bridge

Of the three equations below, one has one solution, one has no solutions, and one has infinitely many solutions. Match each equation with the number of solutions it has.[1]

1. $12(x-3)+18=6(2x-3)$
2. $12(x-3)+18=4(3x-3)$
3. $12(x-3)+18=4(2x-3)$

a. one solution

b. no solutions

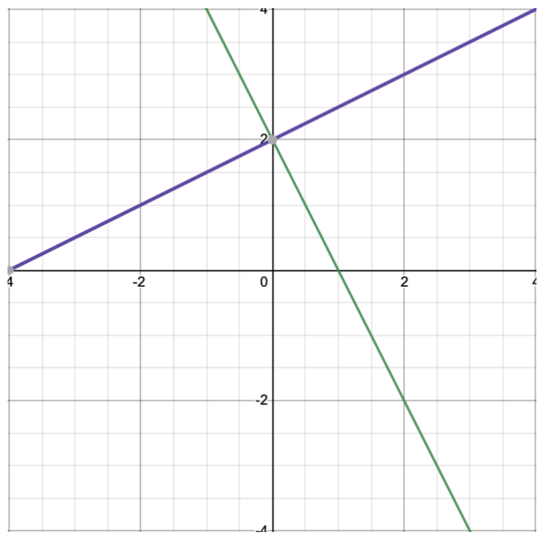
c. infinitely many solutions

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

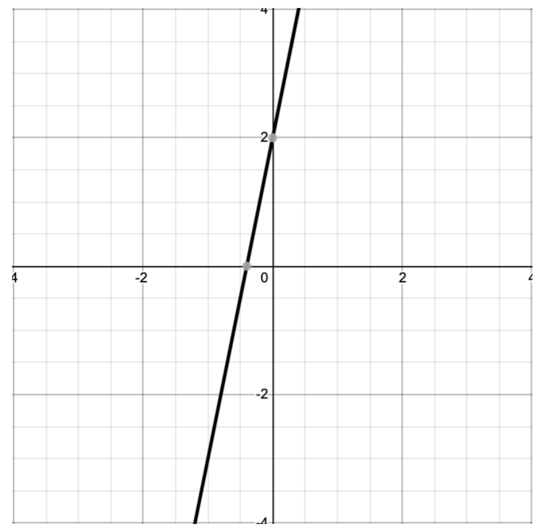
Warm-up: Graphs of Systems

Which one doesn’t belong? Explain your reasoning.

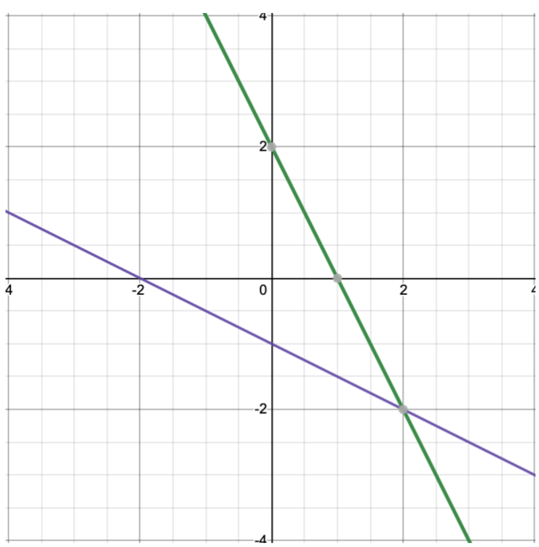
a.



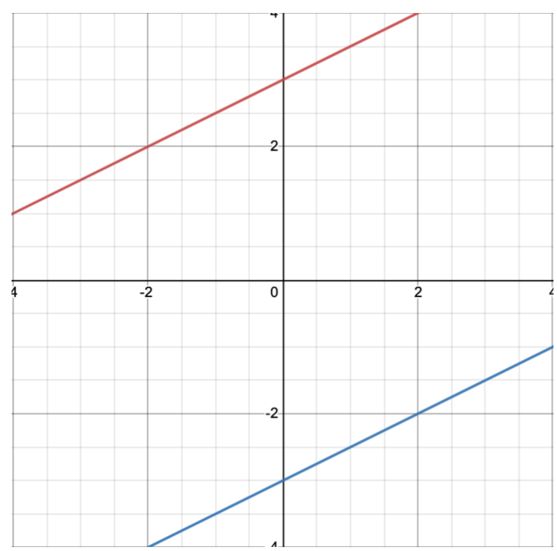
b.



c.



d.



Activity 1: What's the Deal?

A recreation center is offering special prices on its pool passes and gym memberships for the summer. On the first day of the offering, a family paid $96 for four pool passes and two gym memberships. Later that day, an individual bought a pool pass for herself, a pool pass for a friend, and one gym membership. She paid $72.

1. Write a system of equations that represents the relationships between pool passes, gym memberships, and the costs. Be sure to state what each variable represents.

2. Find the price of a pool pass and the price of a gym membership by solving the system algebraically. Explain or show your reasoning.

3. Use graphing technology to graph the equations in the system. Make 1–2 observations about your graphs.

Activity 2: Sorting Systems

Your teacher will give you a set of cards. Each card contains a system of equations.

Sort the systems into three groups based on the number of solutions each system has. Be prepared to explain how you know where each system belongs.

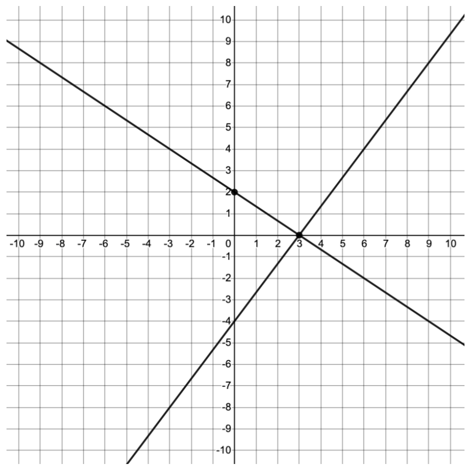
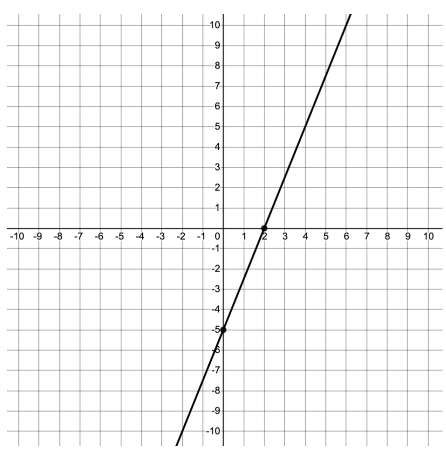
Are You Ready For More?

1. In the cards, for each system with no solution, change a single constant term so that there are infinitely many solutions to the system.

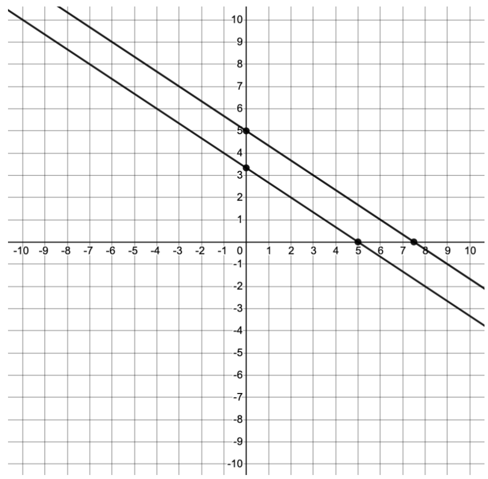
2. For each system with infinitely many solutions, change a single constant term so that there are no solutions to the system.

3. Explain why in these situations it is impossible to change a single constant term so that there is exactly one solution to the system.

Lesson Debrief

Number of Solutions	**Graphs** What are some characteristics of the graph of the equations in the system?	**Algebraic Solutions** What do we get when solving the system of equations algebraically?
a. ________	 Characteristics of Graph: ____________________ ____________________	$\begin{cases} 4x - 3y = 12 \\ 2x + 3y = 6 \end{cases}$ Solve algebraically: $4x - 3y = 12$ $+(2x + 3y = 6)$ __________ $6x = 18$ $x = 3$ When $x = 3$, $4(3) - 3y = 12$ $12 - 3y = 12$ $-3y = 0$ $y = 0$ Therefore, the solution is (3, 0) Characteristics of Equations of System: ____________________ ____________________
b. ________	 Characteristics of Graph: ____________________ ____________________	$\begin{cases} 5x - 2y = 10 \\ x - \frac{2}{5}y = 2 \end{cases}$ $5x - 2y = 10$ $5(x - \frac{2}{5}y = 2)$ $\longrightarrow$ $5x - 2y = 10$ $-(5x - 2y = 10)$ __________ $0 = 0$ Characteristics of Equations of System: ____________________ ____________________

c. ________



Characteristics of Graph:

____

____

$$\begin{cases} 2x + 3y = 15 \\ 4x + 6y = 20 \end{cases}$$

$$2(2x + 3y = 15) \longrightarrow \begin{array}{r} 4x + 6y = 30 \\ -4x + 6y = 20 \\ \hline 0 \neq 10 \end{array}$$

Characteristics of Equation of System:

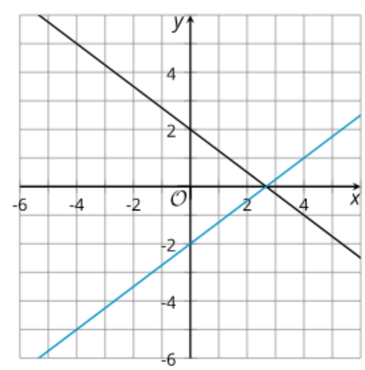
____

____

Lesson 12 Summary and Glossary

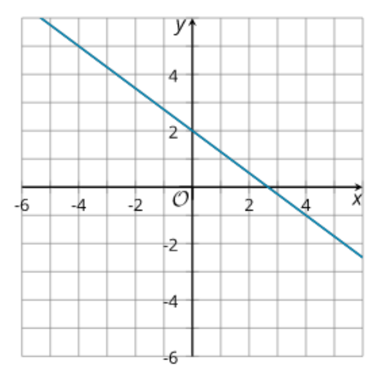
We have seen many examples of a system where one pair of values satisfies both equations. Not all systems, however, have one solution. Some systems have many solutions, and others have no solutions.

Let's look at three systems of equations and their graphs.



System 1: $\begin{cases} 3x + 4y = 8 \\ 3x - 4y = 8 \end{cases}$

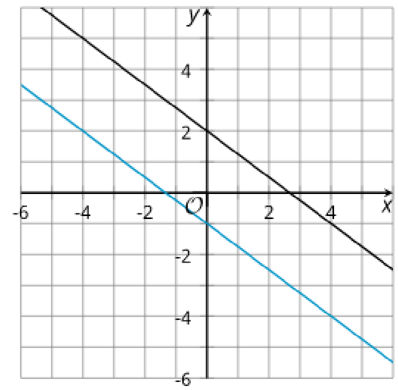
The graphs of the equations in system 1 intersect at one point. The coordinates of the point are the one pair of values that are simultaneously true for both equations. When we solve the equations, we get exactly one solution.



System 2: $\begin{cases} 3x + 4y = 8 \\ 6x + 8y = 16 \end{cases}$

The graphs of the equations in system 2 appear to be the same line. This suggests that every point on the line is a solution to both equations, or that the system has infinitely many solutions.

System 3: $\begin{cases} 3x + 4y = 8 \\ 3x + 4y = -4 \end{cases}$



The graphs of the equations in system 3 appear to be parallel. If the lines never intersect, then there is no common point that is a solution to both equations and the system has no solutions.

How can we tell, without graphing, that system 2 indeed has many solutions?

- Notice that $3x + 4y = 8$ and $6x + 8y = 16$ are equivalent equations. Multiplying the first equation by 2 gives the second equation. Multiplying the second equation by $\frac{1}{2}$ gives the first equation. This means that any solution to the first equation is a solution to the second.
- Rearranging $3x + 4y = 8$ into slope-intercept form gives $y = \frac{8-3x}{4}$, or $y = 2 - \frac{3}{4}x$. Rearranging $6x + 8y = 16$ gives $y = \frac{16-6x}{8}$, which is also $y = 2 - \frac{3}{4}x$. Both lines have the same slope and the same y-value for the vertical intercept!

How can we tell, without graphing, that system 3 has no solutions?

- Notice that in one equation $3x + 4y$ equals 8, but in the other equation it equals -4. Because it is impossible for the same expression to equal 8 and -4, there must not be a pair of x- and y-values that are simultaneously true for both equations. This tells us that the system has no solutions.
- Rearranging each equation into slope-intercept form gives $y = 2 - \frac{3}{4}x$ and $y = -1 - \frac{3}{4}x$. The two graphs have the same slope but the y-values of their vertical intercepts are different. This tells us that the lines are parallel and will never cross.

Unit 3 Lesson 12 Practice Problems

1. Here is a system of equations: $\begin{cases} 3x - y = 17 \\ x + 4y = 10 \end{cases}$

    a. Solve the system by graphing the equations (by hand or using technology).

    b. Explain how you could tell, without graphing, that there is only one solution to the system.

2. Consider this system of linear equations: $\begin{cases} y = \frac{4}{5}x - 3 \\ y = \frac{4}{5}x + 1 \end{cases}$

    a. Without graphing, determine how many solutions you would expect this system of equations to have. Explain your reasoning.

    b. Try solving the system of equations algebraically and describe the result that you get. Does it match your prediction?

3. How many solutions does this system of equations have? Explain how you know.

$$\begin{cases} 9x - 3y = -6 \\ 5y = 15x + 10 \end{cases}$$

.

4. Select **all** systems of equations that have no solutions.

a. $\begin{cases} y = 5 - 3x \\ y = -3x + 4 \end{cases}$

b. $\begin{cases} y = 4x - 1 \\ 4y = 16x - 4 \end{cases}$

c. $\begin{cases} 5x - 2y = 3 \\ 10x - 4y = 6 \end{cases}$

d. $\begin{cases} 3x + 7y = 42 \\ 6x + 14y = 50 \end{cases}$

e. $\begin{cases} y = 5 + 2x \\ y = 5x + 2 \end{cases}$

5. Here is a system of equations:

$$\begin{cases} -x + 6y = 9 \\ x + 6y = -3 \end{cases}$$

Would you rather use subtraction or addition to solve the system? Explain your reasoning.

(From Unit 3, Lesson 10)

6. Solve each system of equations without graphing.

a. $\begin{cases} 2v + 6w = -36 \\ 5v + 2w = 1 \end{cases}$

b. $\begin{cases} 6t - 9u = 10 \\ 2t + 3u = 4 \end{cases}$

(From Unit 3, Lesson 11)

7. Here is a system of linear equations:

$$\begin{cases} 6x - y = 18 \\ 4x + 2y = 26 \end{cases}$$

Select **all** the steps that would help to eliminate a variable and enable solving.

a. Multiply the first equation by 2, then subtract the second equation from the result.

b. Multiply the first equation by 4 and the second equation by 6, then subtract the resulting equations.

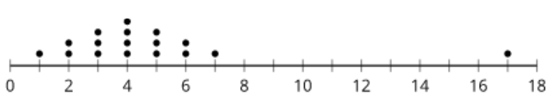
c. Multiply the first equation by 2, then add the result to the second equation.

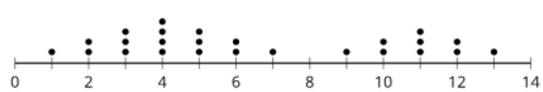
d. Divide the second equation by 2, then add the result to the first equation.

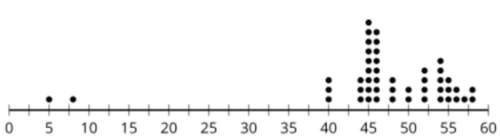
e. Multiply the second equation by 6, then subtract the result from the first equation.

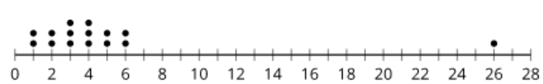
(From Unit 3, Lesson 11)

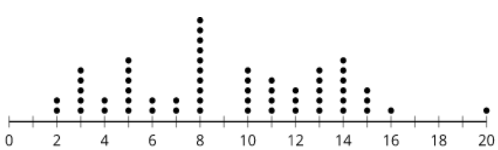
8. Select **all** the dot plots that appear to contain outliers.

a. 

b. 

c. 

d. 

e. 

(From Unit 1)

9. Lin was looking at the equation $2x - 32 + 4(3x - 2462) = 14x$.

   She said, "I can tell right away there are no solutions, because on the left side, you will have $2x + 12x$ and a bunch of constants, but you have just $14x$ on the right side."

   Do you agree with Lin? Explain your reasoning.[2]

   (Addressing NC.8.EE.8)

10. Han was looking at the equation $6x - 4 + 2(5x + 2) = 16x$.

    He said, "I can tell right away there are no solutions, because on the left side, you will have $6x + 10x$ and a bunch of constants, but you have just $16x$ on the right side."

    Do you agree with Han? Explain your reasoning.[3]

    (Addressing NC.8.EE.8)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

[3] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html (see above).

Lessons 13 & 14: Checkpoint

Learning Targets

- I can find the midpoint between two points on a coordinate graph.
- I can find an endpoint of a line if I am given the midpoint and the other endpoint.

Station B: What's the (Mid)point?

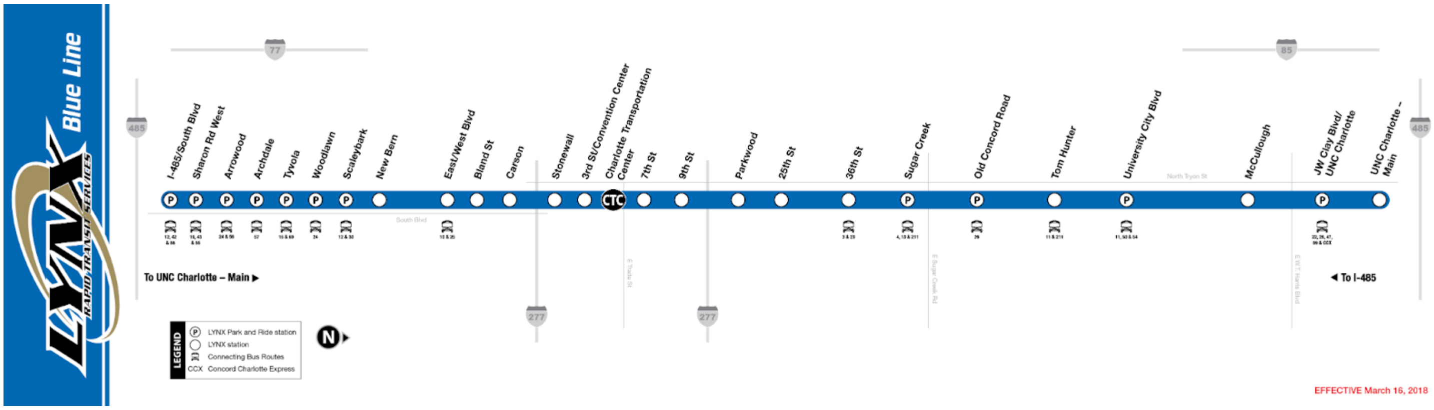
Follow your teacher's directions to access this station's Desmos activity. Use the available space below to show your work.

Station D: One, No, Infinitely Many

Find a partner. Solve question 1 a-c independently. When completed, compare your responses with your partner and answer part d together. Repeat for questions 2 and 3.

	a. Create a second equation that would make a system of equations with one solution:	b. Create a second equation that would make a system of equations with no solution:	c. Create a second equation that would make a system of equations with infinitely many solutions:	d. Compare your equations with your partner. Did you create the same equations? If not, what was different? What was similar? Were you both correct? How do you know?
1. $y = -3x - 12$				
2. $5x - 2y = 10$				
3. $-4x - 2y = 10$				

Station E: Systems in Context



	Adult	Seniors 62+	ADA-disabled	Student K-12
One-way tickets	$2.20	$1.10	$1.10	$1.10
Weekly unlimited pass	$30.80	$30.80	$30.80	$30.80
Monthly unlimited pass	$88.00	$44.00	$44.00	$88.00
10-Ride	$22.00	$9.35	$9.35	$22.00

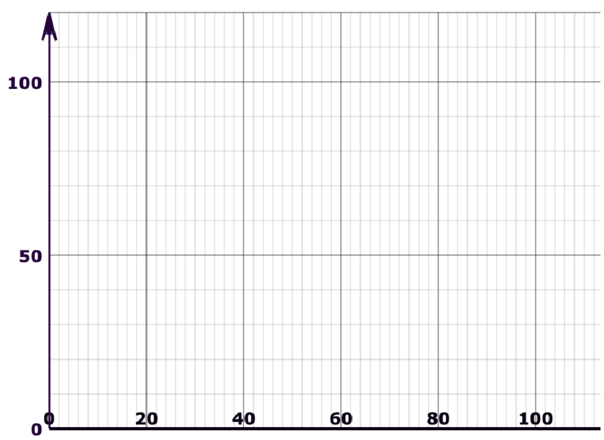
1. It costs $88.00 for a monthly unlimited pass to ride the Light Rail. Without the pass, it costs $2.20 per ride for adults and $1.10 per ride for students.

    a. How much does it cost an adult to ride the Light Rail one time? Five times? Twenty times? Fifty times? Try at least three more values.

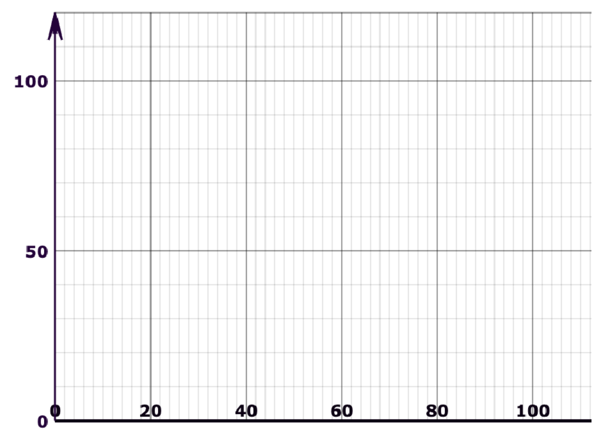
b. How much does it cost a student to ride the Light Rail one time? Five times? Twenty times? Fifty times? Try at least three more values.

c. Construct a graph to compare the costs of the monthly pass and individual one-way tickets for adults and a graph to compare the costs of the monthly pass and individual one-way tickets for students.

Monthly pass



Individual one-way tickets

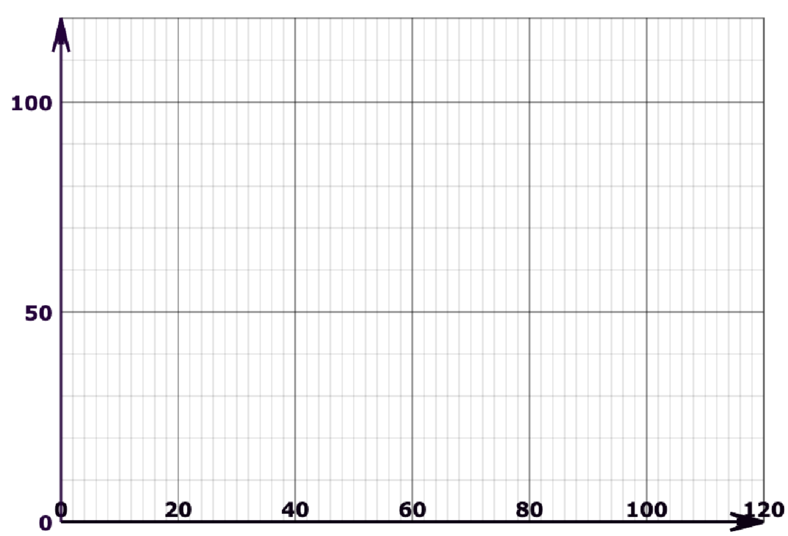


d. Determine how many times an adult would have to ride the light rail before buying the monthly pass is a better deal.

e. Determine how many times a student would have to ride the light rail before buying the monthly pass is a better deal.

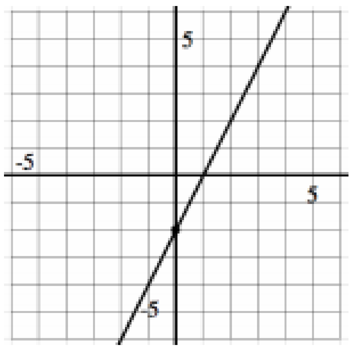
f. Write a description of how you might convince someone whether they should or should not buy a monthly pass.

2. A family came to visit Charlotte on vacation and since they are not staying long, they choose to only buy one-way tickets. The family (which consists of only adults and children in grades K–12) bought 15 one-way tickets that cost them a total of $23.10. Write a system of equations and solve to determine how many adult tickets and how many student tickets they bought.



Station F: Parallel and Perpendicular Lines[1]

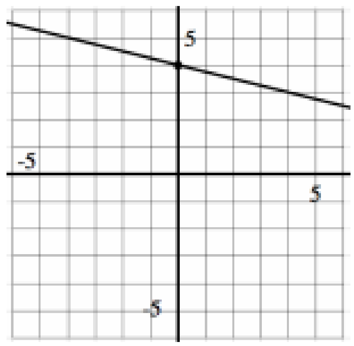
1. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

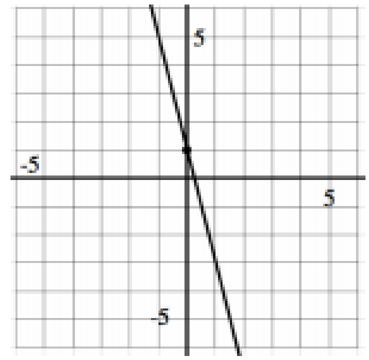
2. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

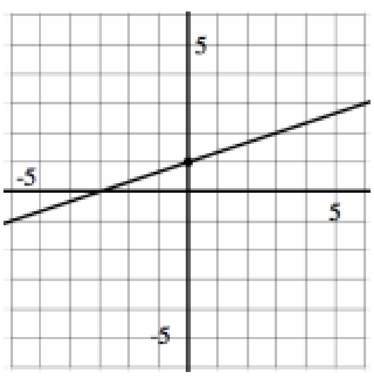
3. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

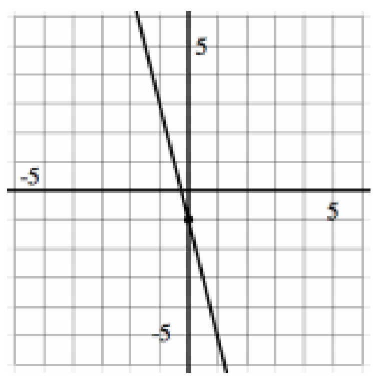
4. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

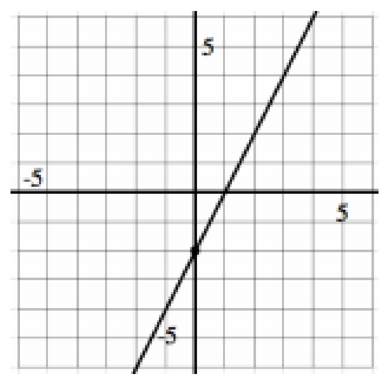
5. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

6. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

[1] Adapted from Secondary Math 1 Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

Station G: Micro-Modeling[2]

Select one of the following questions. Spend about 10 minutes writing your response. Leave your draft response on the table when it is ready for feedback, and pick up another student's draft to review and provide feedback. Note any parts of the writing that are clear, any parts that are confusing, and any parts that seem unfinished. Give feedback on sticky notes or use different colored sticky arrows so that the original student's work doesn't get messed up. After someone has given your draft feedback, use the remaining time to improve your response.

1. The following question was posted on an internet gardening forum:

   I am in charge of purchasing soil for my neighborhood's community garden and I am trying to figure out how many yards of soil I need but have no idea how much a yard of soil actually is. The nursery says they deliver 6 yards in a dump truck. I realize that a yard is 3 feet. But what is a yard of soil—is that 3 feet long and 3 feet high? I'm clueless! LOL

   Write a helpful response to the person who posted this question.

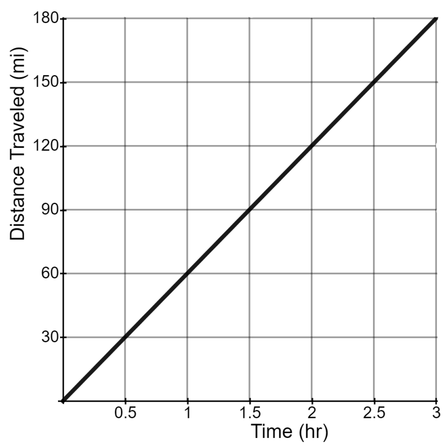
[2] Adapted from Achievethecore.org

2. Each graph below shows the relationship between distance traveled and time for a different train (Train A, Train B, and Train C).

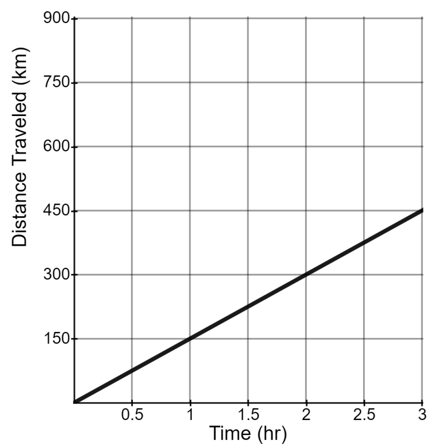
   Which train was traveling fastest during the interval of time shown? Justify your answer with a thorough explanation using words, numbers, and/or visuals.

   Distance Traveled Vs. Time

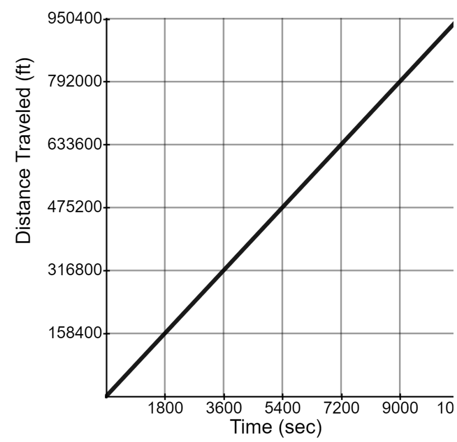
Train A



Train B



Train C



Lesson 15: Graphing Linear Inequalities in Two Variables (Part One)

Learning Targets

- I can find the solutions to a two-variable inequality by using the graph of a related two-variable equation.
- Given a two-variable inequality that represents a situation, I can interpret points in the coordinate plane and decide if they are solutions to the inequality.
- I can write inequalities to describe the constraints in a situation.

Bridge

Mai is at an amusement park. She bought 14 tickets, and each ride requires 2 tickets.[1]

a. Write an expression that gives the number of tickets Mai has left in terms of x, the number of rides she has already gone on. Find at least one other expression that is equivalent to it.

b. $14 - 2x$ represents the number of tickets Mai has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

- 14
- -2
- $2x$

c. $2(7 - x)$ also represents the number of tickets Mai has left after she has gone on x rides. How can each of the following numbers and expressions be interpreted in terms of tickets and rides?

- 7
- $(7 - x)$
- 2

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

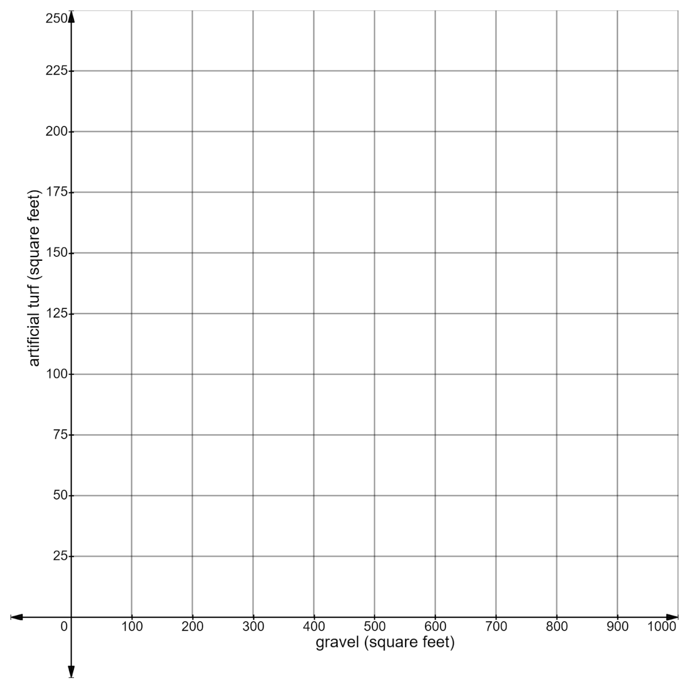
Warm-up: Landscaping Options

A homeowner is making plans to landscape her yard to make it more low-maintenance. She plans to hire professionals to install artificial turf in some parts of the yard and gravel in other parts.

Artificial turf costs $15 per square foot to install and gravel costs $3 per square foot to install. She may use a combination of the two materials in different parts of the yard. Her budget is $3,000.

1. Write an equation that represents the square feet of gravel, x, and the square feet of artificial turf, y, that she could afford if she used her entire budget.

2. On the coordinate plane, sketch a graph that represents your equation. Be prepared to explain your reasoning.



3. What does the point (500,100) mean?

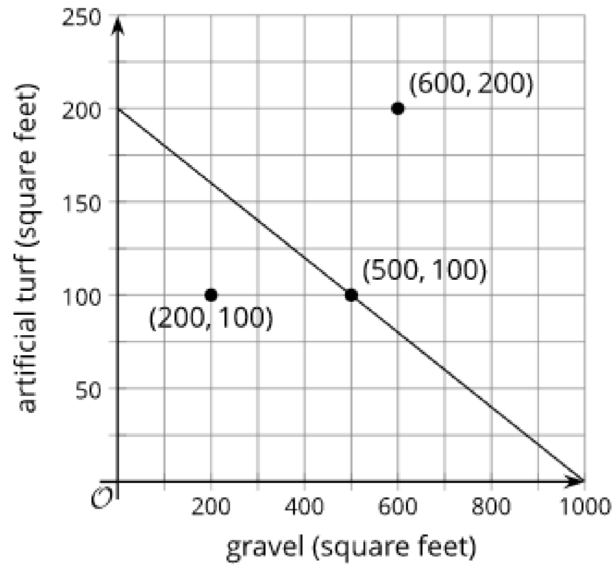
4. What do the solutions to the equation of the line you graphed mean?

Activity 1: Rethinking Landscaping

In the previous problem, we encountered a homeowner looking to update her landscape to include low-maintenance materials.

She is considering artificial turf, which costs $15 per square foot to install, and gravel, which costs $3 per square foot. She may use a combination of the two materials in different parts of the yard. Her budget is still $3,000.

Here is the graph representing some of the constraints in this situation once again.



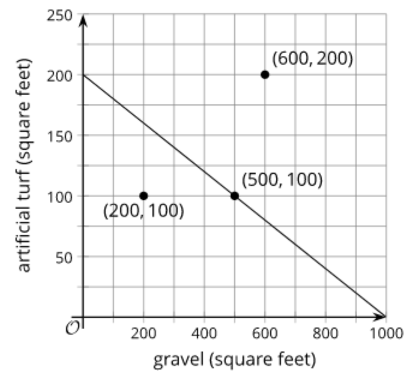
1. The point $(600, 200)$ is located to the right and above the line.

   a. Does that combination of turf and gravel meet the homeowner's constraints? Explain or show your reasoning.

   b. Choose another point in the same region (to the right and above the line). Check if the combination meets the homeowner's constraints.

   c. What do the points above the line represent?

2. The point $(200, 100)$ is located to the left and below the line.



   a. Does that combination of turf and gravel meet the homeowner’s constraints? Explain or show your reasoning.

   b. Choose another point in the same region (to the left and below the line). Check if the combination meets the homeowner’s constraints.

   c. What do the points below the line represent?

3. Write an inequality that represents the constraints in this situation. Explain what the solutions to this inequality would mean.

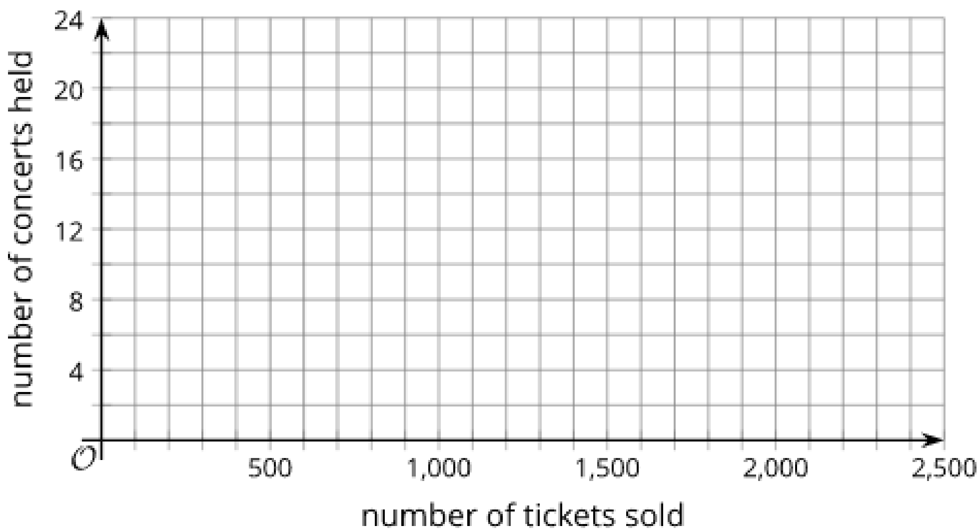
Activity 2: Charity Concerts

A popular band is trying to raise at least $20,000 for charity by holding multiple concerts at a park. It plans to sell tickets at $25 each. For each two-hour concert, the band would need to pay the park $1,250 in fees for security, cleaning, and traffic services.

The band needs to find the combinations of total number of tickets sold, t, and number of concerts held, c, that would allow it to reach its fundraising goal.

1. Write an equation to represent raising **exactly** $20,000 for charity.

2. Graph the line containing the solutions to your equation on the coordinate plane.



3. Name two possible combinations of number of tickets sold and number of concerts held that would allow the band to meet its goal. Plot these ordered pairs on the graph above.

4. Which combination of tickets and concerts would mean **more** money for charity:

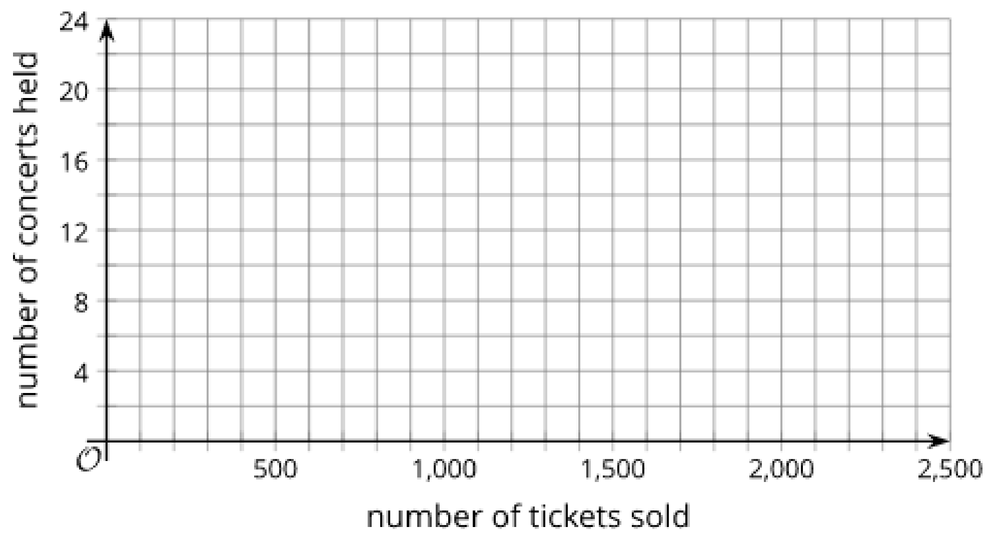
   a. 1,300 tickets and 10 concerts, or 1,300 tickets and 5 concerts?

   b. 1,600 tickets and 16 concerts, or 1,200 tickets and 9 concerts?

   c. 2,000 tickets and 4 concerts, or 2,500 tickets and 10 concerts?

5. Write the inequality that represents raising **at least** $20,000 for charity.

6. Using the combinations of tickets and concerts presented in question 4, plot the ordered pairs that would satisfy the inequality in question 5 on the graph.



Lesson Debrief

Lesson 15 Summary and Glossary

Inequalities in two variables can represent constraints in real-life situations. Graphing their solutions can enable us to solve problems.

Suppose a café is purchasing coffee and tea from a supplier and can spend up to $\$1,000$. Coffee beans cost $\$12$ per kilogram and tea leaves cost $\$8$ per kilogram.

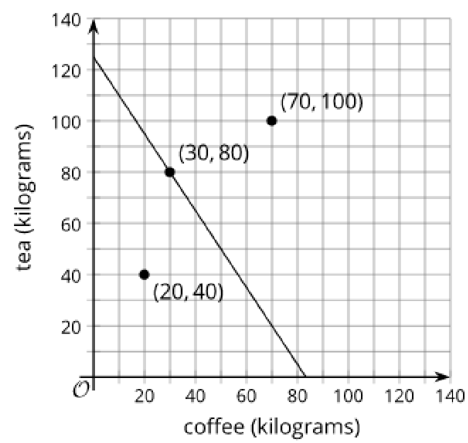
Buying c pounds of coffee beans and t pounds of tea leaves will therefore cost $12c + 8t$. To represent the budget constraints, we can write: $12c + 8t \le 1,000$.

The solution to this inequality is any pair of c and t that makes the inequality true. In this situation, it is any combination of the pounds of coffee and tea that the café can order without going over the $\$1,000$ budget.

We can try different pairs of c and t to see what combinations satisfy the constraint, but it would be difficult to capture all the possible combinations this way. Instead, we can graph a related equation, $12c + 8t = 1,000$, and then find out which region represents all possible solutions.

Here is the graph of that equation.

To determine the solution region, let's take one point on the line and one point on each side of the line, and see if the pairs of values produce true statements.



A point on the line: $(30, 80)$

$$12(30) + 8(80) \le 1,000$$
$$360 + 640 \le 1,000$$
$$1,000 \le 1,000$$

This is true.

A point below the line: $(20, 40)$

$$12(20) + 8(40) \le 1,000$$
$$240 + 320 \le 1,000$$
$$560 \le 1,000$$

This is true.

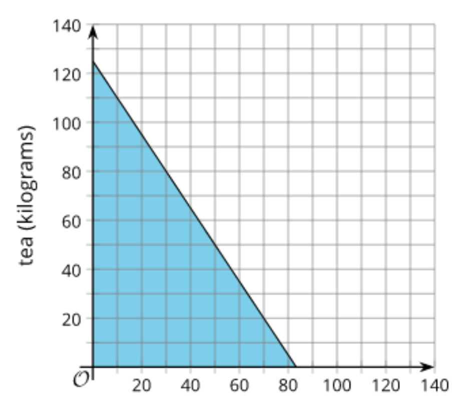
A point above the line: $(70, 100)$

$$12(70) + 8(100) \le 1,000$$
$$840 + 800 \le 1,000$$
$$1,640 \le 1,000$$

This is false.

The points on the line and in the region below the line are solutions to the inequality. Let's shade the solution region.

It is easy to read solutions from the graph. For example, without any computation, we can tell that $(50, 20)$ is a solution because it falls in the shaded region. If the café orders 50 kilograms of coffee and 20 kilograms of tea, the cost will be less than $\$1,000$.



Unit 3 Lesson 15 Practice Problems

1. To qualify for a loan from a bank, the total in someone's checking and savings accounts together must be $500 or more.

   a. Find three possible combinations of checking and savings account balances that would together amount to $500 or more.

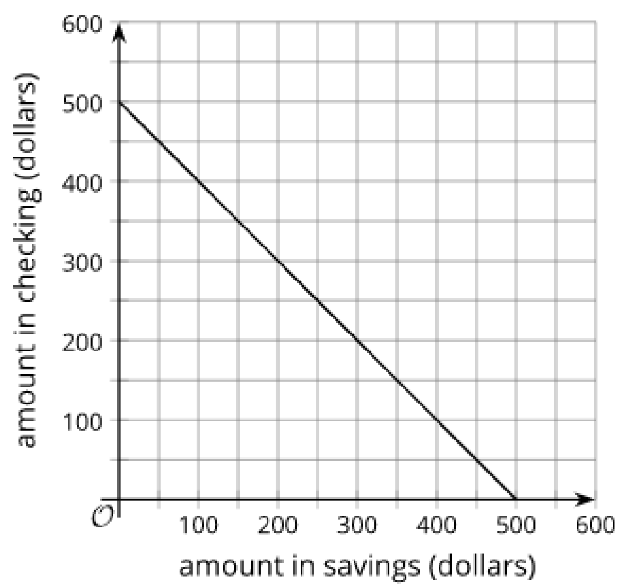
   b. Would someone qualify for a loan if they have $50 in savings and $450 in checking? Why?

   c. Which of these inequalities **best** represents this situation?

   - $x + y < 500$
   - $x + y \leq 500$
   - $x + y > 500$
   - $x + y \geq 500$

   d. To the right a graph of the line $x + y = 500$. Plot ordered pairs representing the savings and checking account combinations you identified in part a and the combination provided in part b on the graph below.

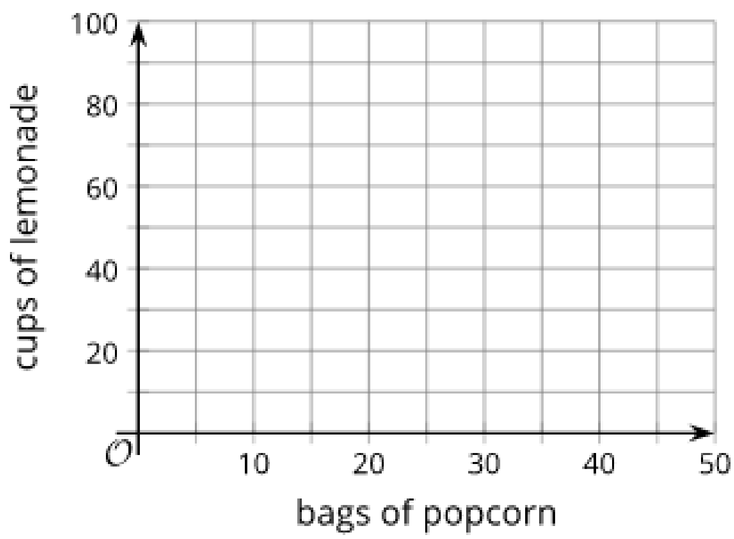
   What general region defines where these points are located relative to the graphed line? (example: above, below, on)



2. The soccer team is selling bags of popcorn for $3 each and cups of lemonade for $2 each. To make a profit, they must collect a total of more than $120.

   a. Write an equation to represent the number of bags of popcorn sold, p, and the number of cups of lemonade sold, c, in order to break even.

   b. Graph the line representing your equation in part a on the coordinate plane.



   c. Explain how we could check if the points above, below, and on the line correspond to the team earning a profit.

   d. Write an inequality to represent the number of bags of popcorn sold, p, and the number of cups of lemonade sold, c, in order to make a profit.

3. Tyler filled a small jar with quarters and dimes and donated it to his school's charity club. The club member receiving the jar asked, "Do you happen to know how much is in the jar?" Tyler said, "I know it's at least $8.50, but I don't know the exact amount."

   a. Write an inequality to represent the relationship between the number of dimes, d, the number of quarters, q, and the dollar amount of the money in the jar.

   b. Identify one solution to this inequality and explain what a solution would mean in this situation.

   c. Suppose Tyler knew there are 25 dimes in the jar. Write an inequality that represents how many quarters could be in the jar.

4. Square $ABCD$ is drawn on the coordinate plane with vertex A at $(-4,-2)$ and the midpoint of side AB at $(-1,-6)$. What is the area of the square?

(From Unit 3, Lessons 13 & 14)

5. Kiran says, “I bought 2.5 pounds of red and yellow lentils. Both were $1.80 per pound. I spent a total of $4.05.”

   a. Write a system of equations to describe the relationships between the quantities in Kiran's statement. Be sure to specify what each variable represents.

   b. Elena says, “That can't be right.” Explain how Elena can tell that something is wrong with Kiran's statement.

   c. Kiran says, “Oops, I meant to say I bought 2.25 pounds of lentils.” Revise your system of equations to reflect this correction.

   d. Is it possible to tell for sure how many pounds of each kind of lentil Kiran might have bought? Explain your reasoning.

(From Unit 3, Lesson 12)

6. Andre is solving the inequality $14x + 3 \leq 8x + 3$. He first solves a related equation.

$$14x + 3 = 8x + 3$$
$$14x = 8x$$
$$8 = 14$$

This seems strange to Andre. He thinks he probably made a mistake. What was his mistake?

(From Unit 2)

7. Here is an inequality: $-7 - (3x + 2) < -8(x + 1)$

Select **all** the values of x that are solutions to the inequality.

a. $x = -0.2$

b. $x = -0.1$

c. $x = 0$

d. $x = 0.1$

e. $x = 0.2$

f. $x = 0.3$

(From Unit 2)

8. Solve this inequality:

   $\frac{x-4}{3} \geq \frac{x+3}{2}$

   (From Unit 2)

9. At a store, a shirt was marked down in price by $\$10.00$. A pair of pants doubled in price. Following these changes, the price of every item in the store was cut in half. Write two different expressions that represent the new cost of the items, using s for the cost of each shirt and p for the cost of a pair of pants. Explain the different information each one shows.[2]

   (Addressing NC.6.EE.2)

[2] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds.

Lesson 16: Graphing Linear Inequalities in Two Variables (Part Two)

Learning Targets

- Given a two-variable inequality and the graph of the related equation, I can determine which side of the line the solutions to the inequality will fall.
- I can describe the graph that represents the solutions to a linear inequality in two variables.

Warm-up: Less Than, Equal to, or More Than 6?

Here is an expression: $-2x+3y$. Decide if the values in each ordered pair, (x,y), make the value of the expression less than, greater than, or equal to 6.

1. $(2,4)$

2. $(0,2)$

3. $(-1,1)$

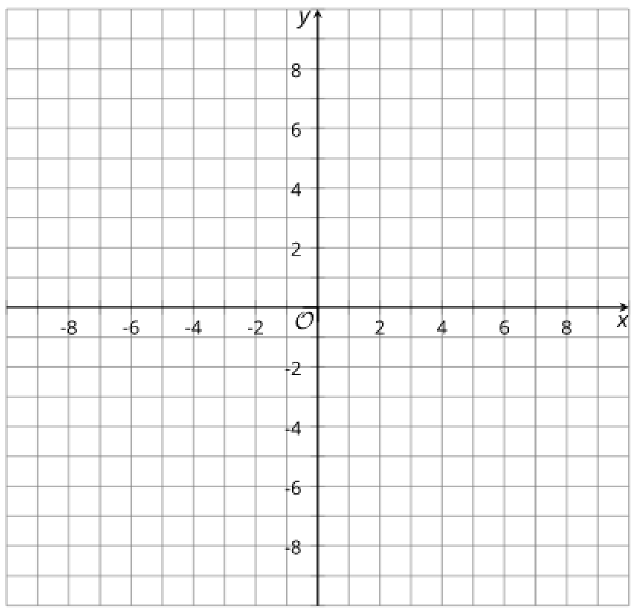
4. $(-2,1)$

Activity 1: Finding the Solution Region

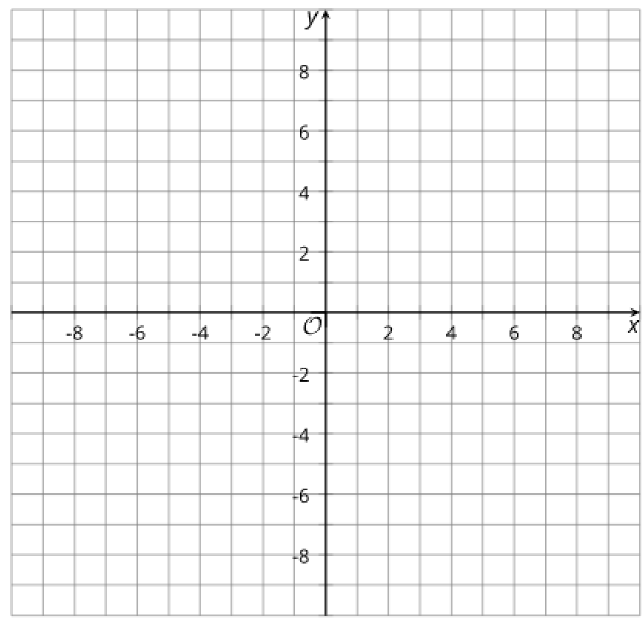
Here are four inequalities. Study each inequality assigned to your group and work with your group to:

- Graph the related equation
- Determine the side of the line that has the solutions to the inequality
- Shade the solution region

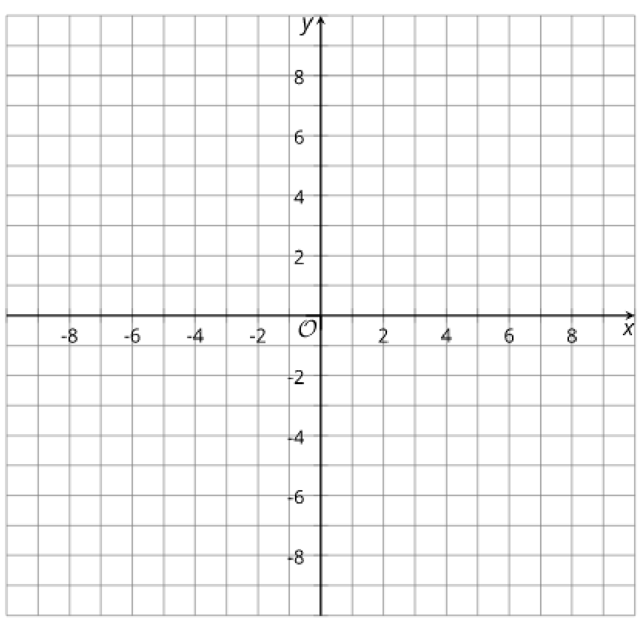
$x \geq y$



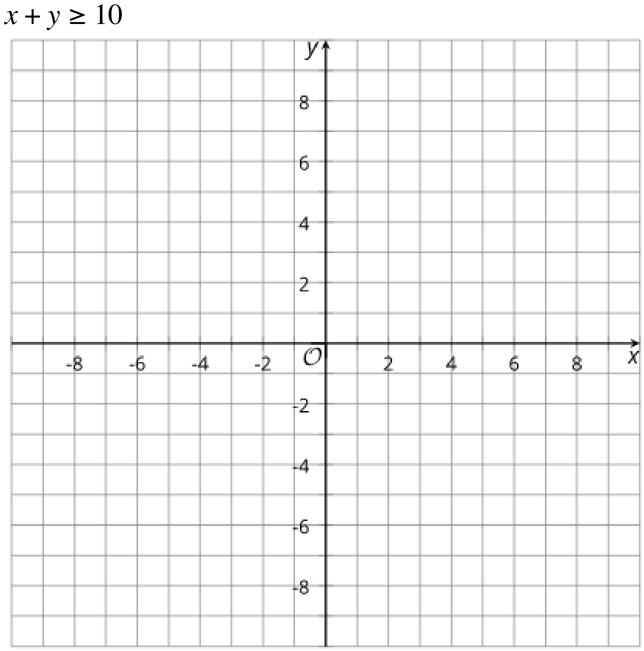
$-2y \geq -4$



$3x \leq 0$

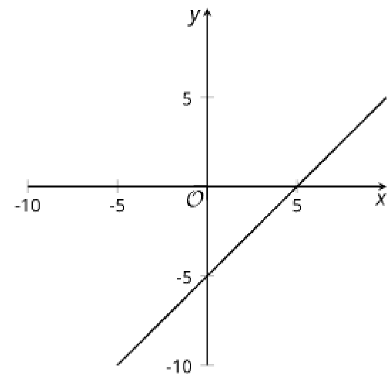


$x + y \geq 10$



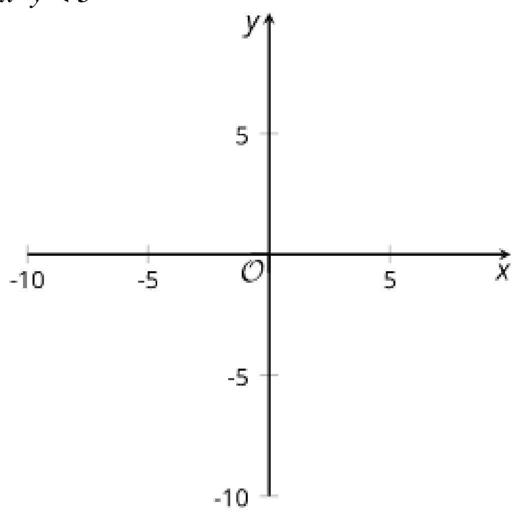
Activity 2: Sketching Solutions to Inequalities

1. Here is a graph that represents solutions to the equation $x - y = 5$.

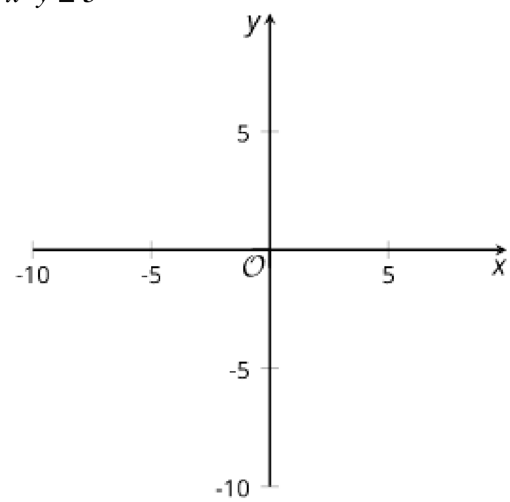


Sketch two quick graphs representing the solutions to each of these inequalities:

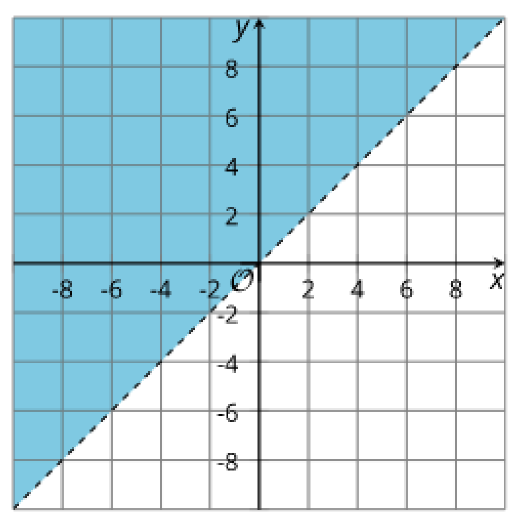
$x - y < 5$



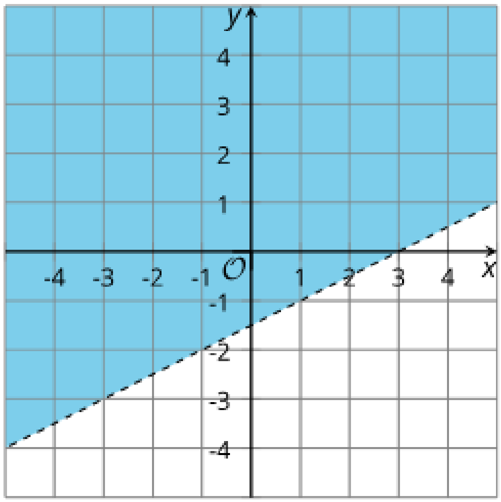
$x - y \geq 5$



2. For the graph below, write an inequality whose solutions are represented by the shaded part of the graph.



Are You Ready For More?



1. The points $(7,3)$ and $(7,5)$ are both in the solution region of the inequality $x-2y<3$.

   a. Compute $x-2y$ for both of these points.

   b. Which point comes closest to satisfying the equation $x-2y=3$? That is, for which (x,y) pair is $x-2y$ closest to 3?

2. The points $(3,2)$ and $(5,2)$ are also in the solution region. Which of these points comes closest to satisfying the equation $x-2y=3$?

3. Find a point in the solution region that comes even closer to satisfying the equation $x-2y=3$. What is the value of $x-2y$?

4. For the points $(5,2)$ and $(7,3)$, $x-2y=1$. Find another point in the solution region for which $x-2y=1$.

5. Find $x-2y$ for the point $(5,3)$. Then find two other points that give the same answer.

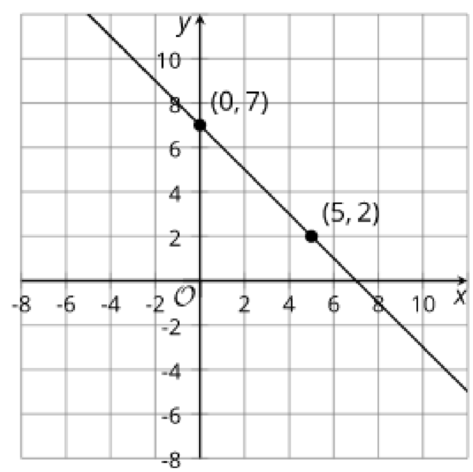
Lesson Debrief

Lesson 16 Summary and Glossary

The equation $x + y = 7$ is an equation in two variables. Its solution is any pair of x and y whose sum is 7. The pairs $x = 0,\ y = 7$ and $x = 5,\ y = 2$ are two examples.

We can represent all the solutions to $x + y = 7$ by graphing the equation on a coordinate plane.

The graph is a line. All the points on the line are solutions to $x + y = 7$.

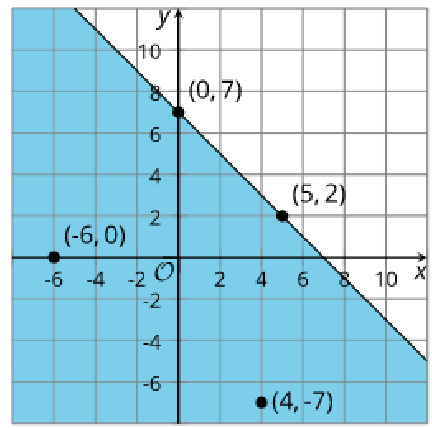


The inequality $x + y \le 7$ is an inequality in two variables. Its solution is any pair of x and y whose sum is 7 or less than 7.

This means it includes all the pairs that are solutions to the equation $x + y = 7$, but also many other pairs of x and y that add up to a value less than 7. The pairs $x = 4,\ y = -7$ and $x = -6,\ y = 0$ are two examples.

On a coordinate plane, the solution to $x+y \le 7$ includes the line that represents $x+y=7$. If we plot a few other (x,y) pairs that make the inequality true, such as $(4,-7)$ and $(-6,0)$, we see that these points fall on one side of the line. (In contrast, (x,y) pairs that make the inequality false fall on the other side of the line.)

We can shade that region on one side of the line to indicate that all points in it are solutions.



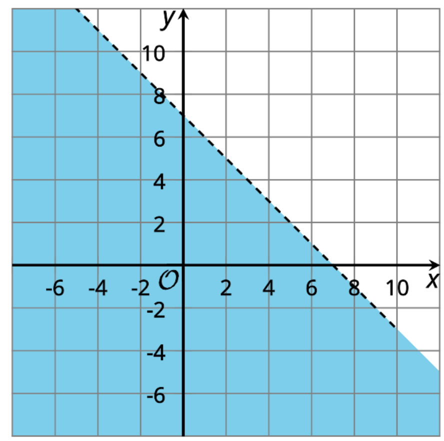
What about the inequality $x+y<7$?

The solution is any pair of x and y whose sum is less than 7. This means pairs like $x=0,\ y=7$ and $x=5,\ y=2$ are *not* solutions.

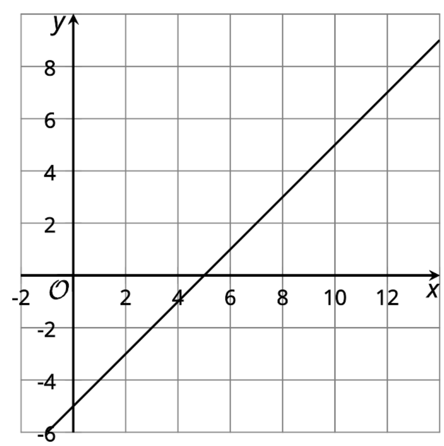
On a coordinate plane, the solution does not include points on the line that represent $x+y=7$ (because those points are x and y pairs whose sum is 7).

To exclude points on that boundary line, we can use a dashed line.

All points below that line are (x,y) pairs that make $x+y<7$ true. The region on that side of the line can be shaded to show that it contains the solutions.



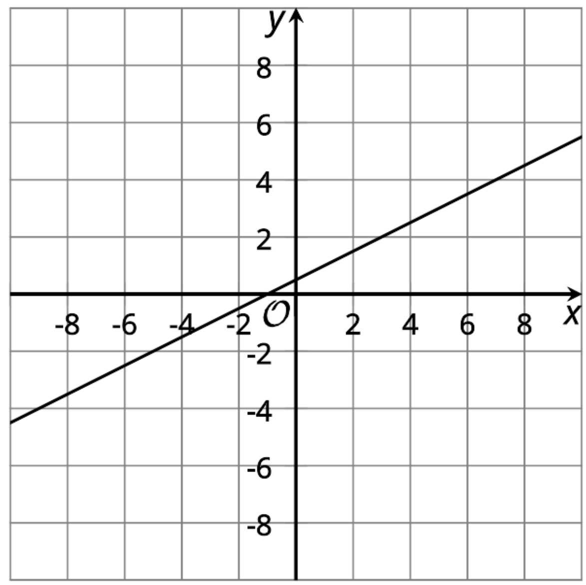
When identifying the solution region, it is important *not* to assume that the solution will be above the line because of a ">" symbol or below the line because of a "<" symbol. For example, when graphing the inequality $x-y \ge 5$ we would start by graphing the related equation $x-y=5$:



Points above the line such as $(0,0)$ are *not* solutions to the inequality because because the (x,y) pairs make the inequality false. Points that are on or below the lines are solutions, so we can shade that lower region.

Unit 3 Lesson 16 Practice Problems

1. Here is a graph of the equation $2y - x = 1$.



   a. Are the points $(0, \frac{1}{2})$ and $(-7, -3)$ solutions to the equation? Explain or show how you know.

   b. Check if each of these points is a solution to the inequality $2y - x > 1$:

   $(0, 2)$ $(8, \frac{1}{2})$ $(-6, 3)$ $(-7, -3)$

   c. Shade the region that represents the solution set to the inequality $2y - x > 1$.

   d. Are the points on the line included in the solution set? Explain how you know.

2. Select **all** coordinate pairs that are solutions to the inequality $5x + 9y < 45$.

   a. $(0, 0)$

   b. $(5, 0)$

   c. $(9, 0)$

   d. $(0, 5)$

   e. $(0, 9)$

   f. $(5, 9)$

   g. $(-5, -9)$

3. Consider the linear equation $2y - 3x = 5$.

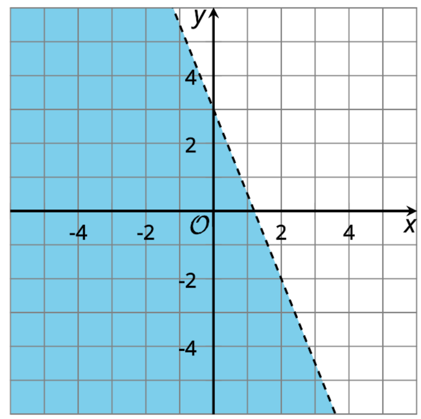
   a. The pair $(-1, 1)$ is a solution to the equation. Find another (x, y) pair that is a solution to the equation.

   b. Are $(-1, 1)$ and $(4, 1)$ solutions to the inequality $2y - 3x < 5$? Explain how you know.

   c. Explain how to use the answers to the previous questions to graph the solution set to the inequality $2y - 3x < 5$.

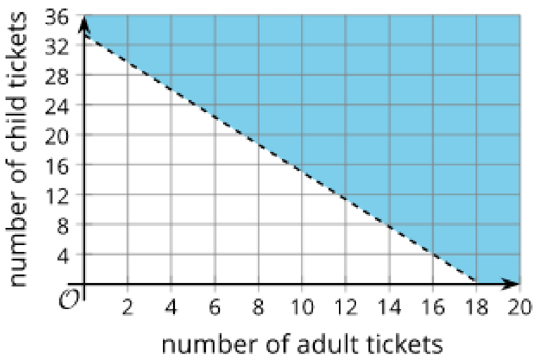
4. The boundary line on the graph represents the equation $5x + 2y = 6$.

   Write an inequality that is represented by the graph.

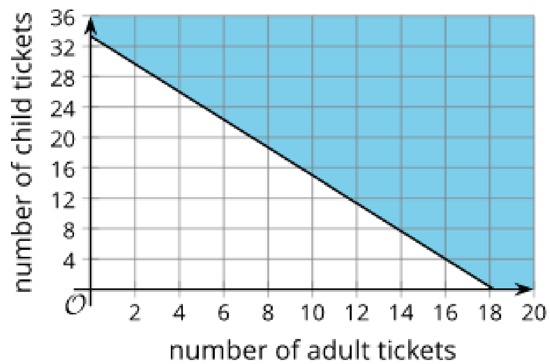


5. Tickets to the aquarium are $11 for adults and $6 for children. An after-school program has a budget of $200 for a trip to the aquarium. If the boundary line in each graph represents the equation $11x + 6y = 200$, which graph represents the cost constraint in this situation?

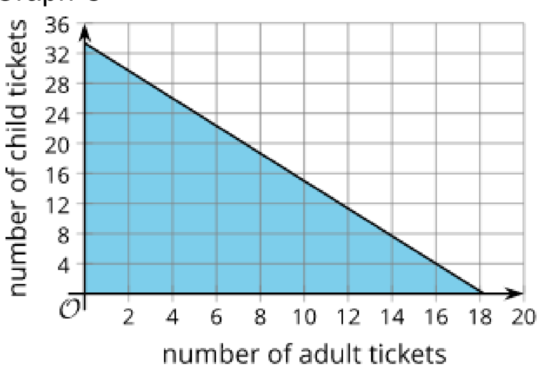
Graph A



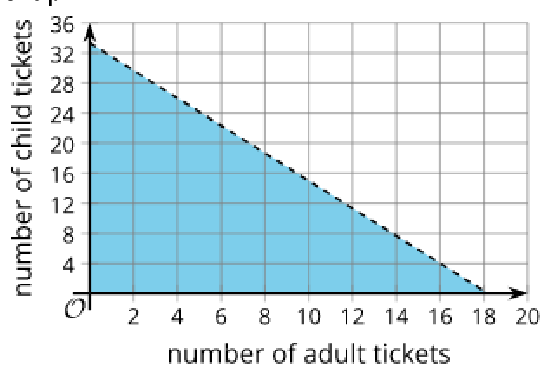
Graph B



Graph C

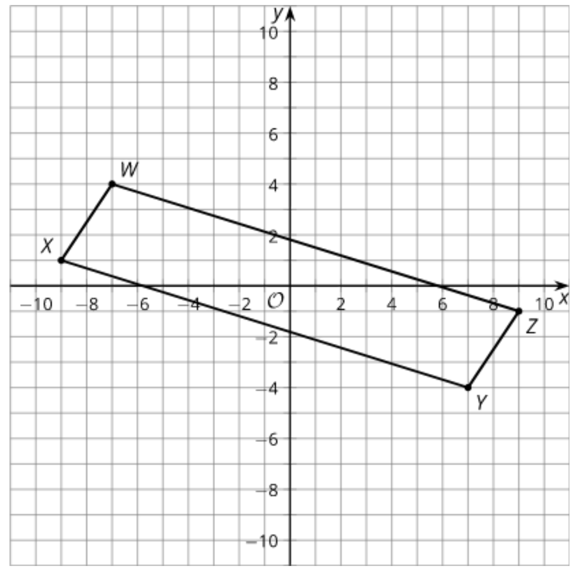


Graph D



6. Diego claims that he can tell, using slopes, that this quadrilateral is a parallelogram. Noah looks at the slopes Diego calculated and says he could be even more specific and call it a rectangle.

   Do you agree with either of them? Explain or show your reasoning.



(From Unit 3, Lesson 8)

7. Solve each system of equations without graphing.

a. $\begin{cases} 4d + 7e = 68 \\ -4d - 6e = -72 \end{cases}$

b. $\begin{cases} \frac{1}{4}x + y = 1 \\ \frac{3}{2}x - y = \frac{4}{3} \end{cases}$

(From Unit 3, Lesson 10)

8. Mai and Tyler are selling items to earn money for their elementary school. The school earns w dollars for every wreath sold and p dollars for every potted plant sold. Mai sells 14 wreaths and 3 potted plants and the school earns $70.50. Tyler sells 10 wreaths and 7 potted plants and the school earns $62.50.

   This situation is represented by this system of equations: $\begin{cases} 14w + 3p = 70.50 \\ 10w + 7p = 62.50 \end{cases}$

   Explain why it makes sense in this situation that the solution of this system is also a solution to $4w + (-4p) = 8.00$.

   (From Unit 3, Lesson 11)

9. Elena is planning to go camping for the weekend and has already spent $40 on supplies. She goes to the store and buys more supplies.

   Which inequality represents d, the total amount in dollars that Elena spends on supplies?

   a. $d > 40$

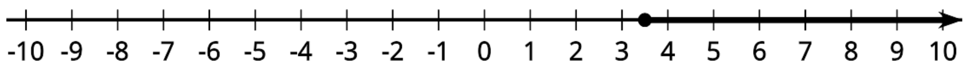
   b. $d \geq 40$

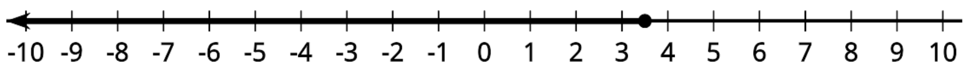
   c. $d < 40$

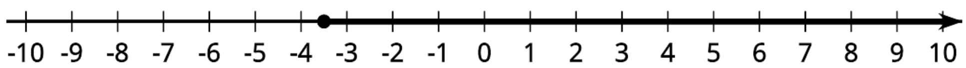
   d. $d \leq 40$

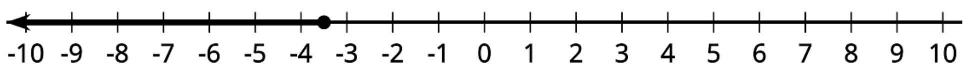
   (From Unit 2)

10. Which graph represents the solution to $\frac{4x-8}{3} \leq 2x - 5$?

a.


b.


c.


d.


(From Unit 2)

11. Solve $-x < 3$. Explain how to find the solution set.

(From Unit 2)

Lesson 17: Solving Problems with Inequalities in Two Variables

Learning Targets

- I can use graphing technology to find the solution to a two-variable inequality.
- When given inequalities, graphs, and descriptions that represent the constraints in a situation, I can connect the different representations and interpret them in terms of the situation.

Bridge[1]

Elena is participating in a fundraiser at school. She will receive donations from two people. A cousin will donate \$0.40 for every $\frac{1}{8}$ mile that Yuri walks. A friend will give Elena a one-time donation of \$30.

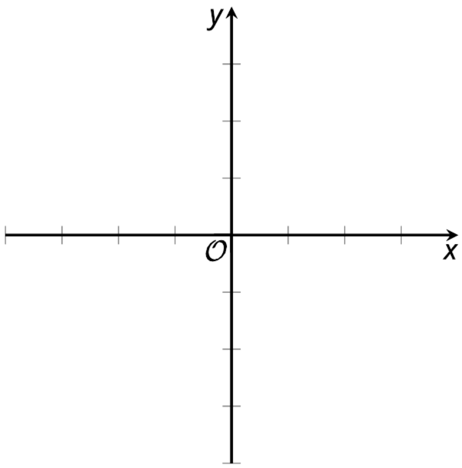
What is the minimum number of miles Elena needs to walk to raise at least \$50?

[1] Adapted from Achievethecore.org

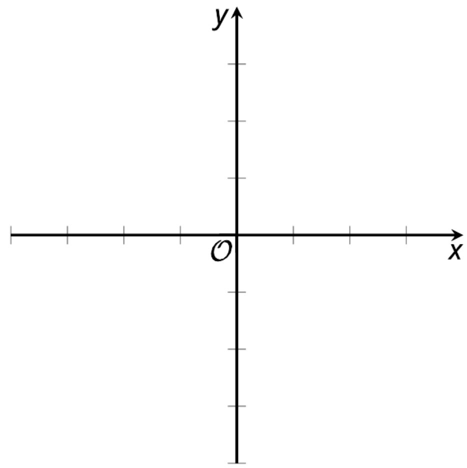
Warm-up: Graphing Inequalities with Technology

Access www.desmos.com/calculator to graph the solution region of each inequality and sketch each graph. Adjust the graphing window as needed to show meaningful information.

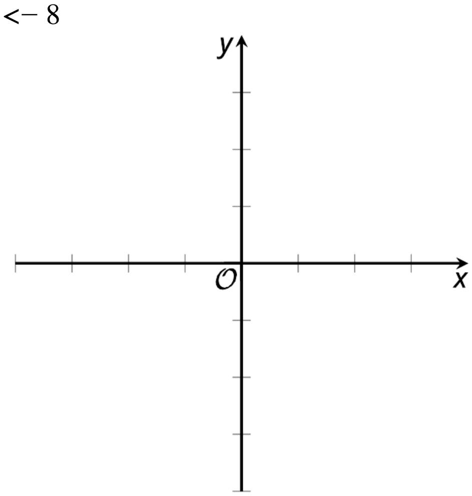
1. $y > x$



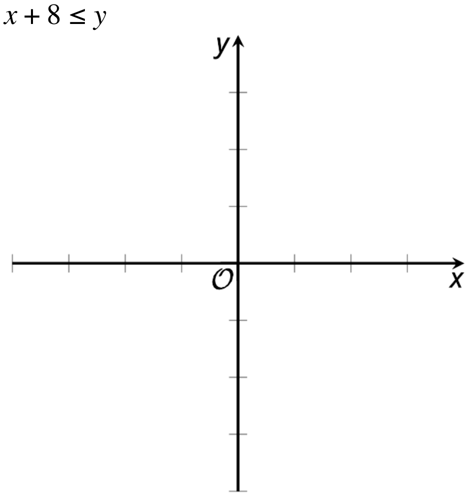
2. $y \geq x$



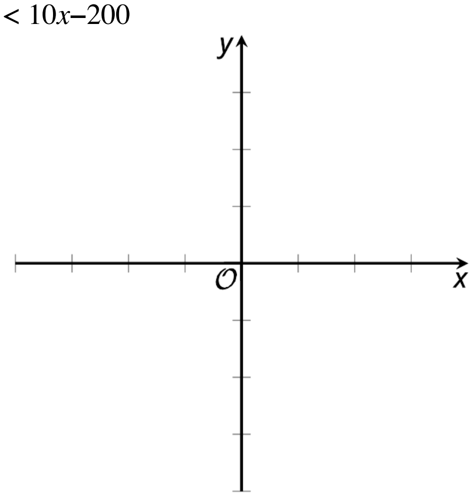
3. $y <- 8$



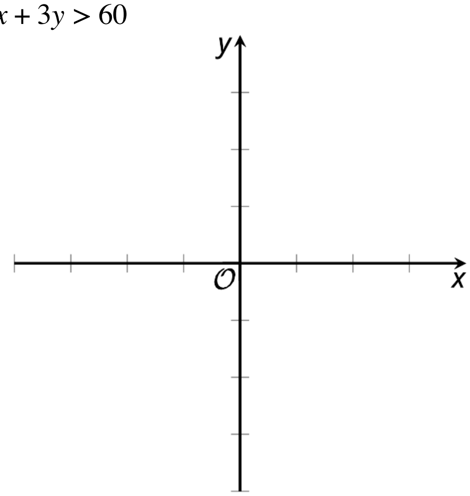
4. $-x + 8 \leq y$



5. $y < 10x - 200$



6. $2x + 3y > 60$



Activity 1: Solving Problems with Inequalities in Two Variables

Here are three situations (bank accounts, concert tickets, and advertising packages). There are two questions about each situation. For each question that you work on:

a. Write an inequality to describe the constraints. Specify what each variable represents.

b. Use Desmos to graph the inequality. Sketch the solution region on the coordinate plane and label the axes.

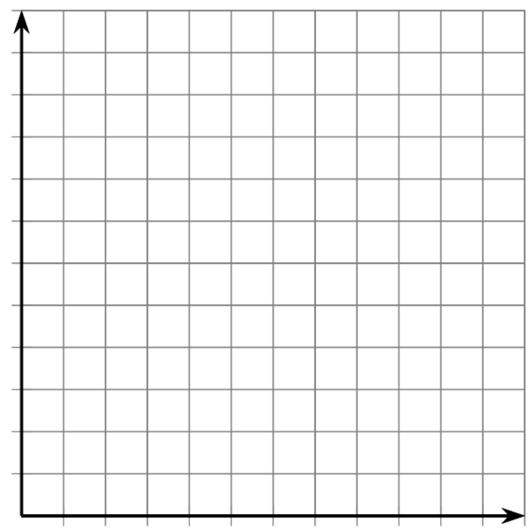
c. Name one solution to the inequality and explain what it represents in that situation.

d. Answer the question about the situation.

Bank Accounts

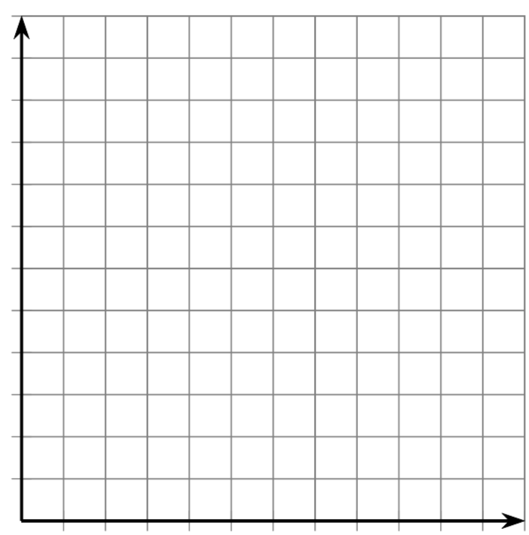
1. A customer opens a checking account and a savings account at a bank. They will deposit a maximum of $600, some in the checking account and some in the savings account. (They might not deposit all of it and instead keep some of the money as cash.)

   If the customer deposits $200 in their checking account, what can you say about the amount they deposit in their savings account?



2. The bank requires a minimum balance of $50 in the savings account. It does not matter how much money is kept in the checking account.

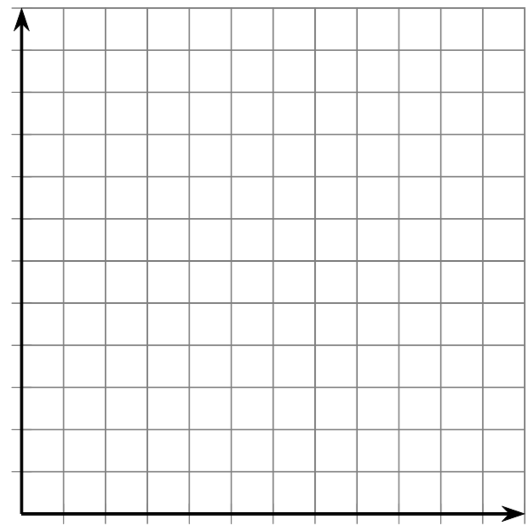
   If the customer deposits no money in the checking account but is able to maintain both accounts without penalty, what can you say about the amount deposited in the savings account?



Concert Tickets

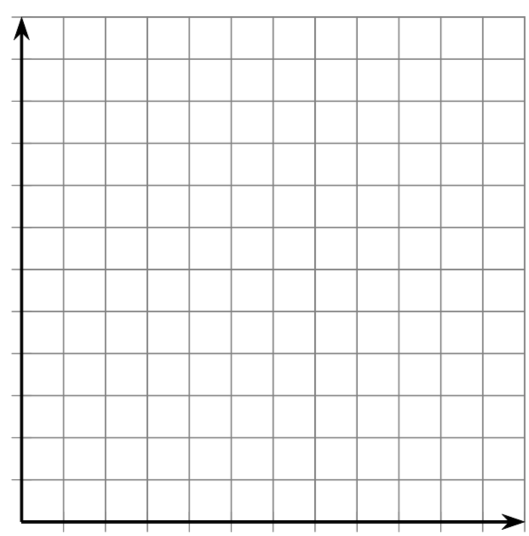
1. Two kinds of tickets to an outdoor concert were sold: lawn tickets and seat tickets. Fewer than 400 tickets in total were sold.

   If you know that exactly 100 lawn tickets were sold, what can you say about the number of seat tickets?



2. Lawn tickets cost \$30 each and seat tickets cost \$50 each. The organizers want to make at least \$14,000 from ticket sales.

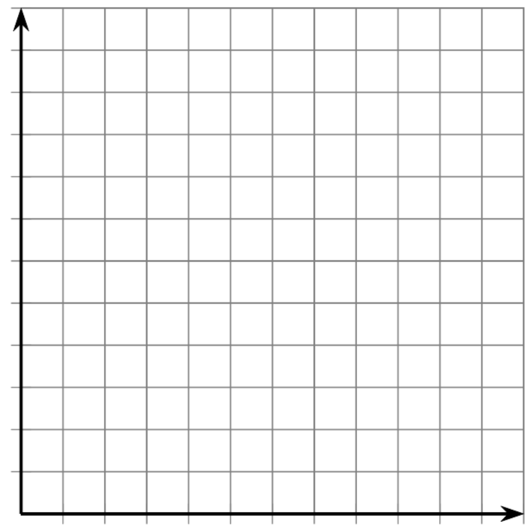
   If you know that exactly 200 seat tickets were sold, what can you say about the number of lawn tickets?



Advertising Packages

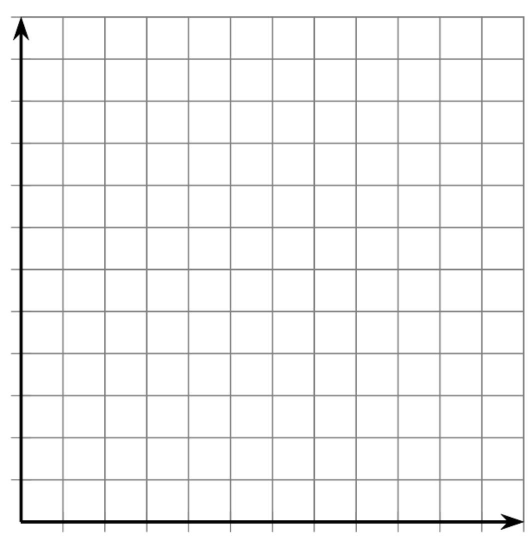
1. An advertising agency offers two packages for small businesses who need advertising services. A basic package includes only design services. A premium package includes design and promotion. The agency's goal is to sell at least 60 packages in total.

   If the agency sells exactly 45 basic packages, what can you say about the number of premium packages it needs to sell to meet its goal?



2. The basic advertising package has a value of $1,000 and the premium package has a value of $2,500. The goal of the agency is to sell more than $60,000 worth of small-business advertising packages.

   If you know that exactly 10 premium packages were sold, what can you say about the number of basic packages the agency needs to sell to meet its goal?

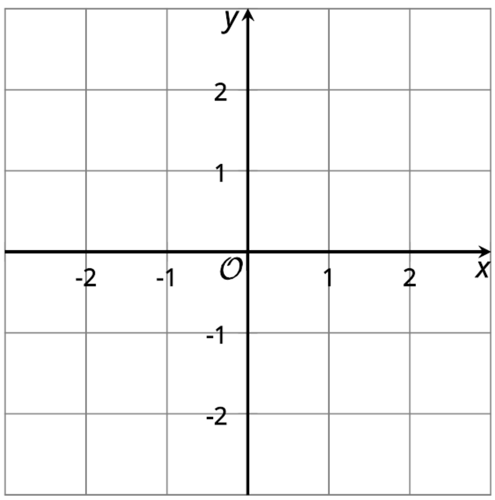


Are You Ready For More?

This activity will require a partner.

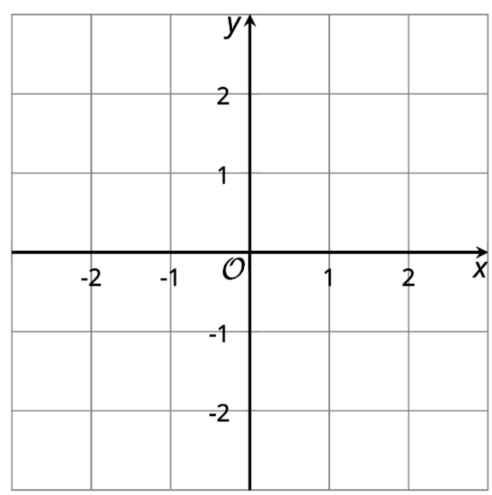
1. Without letting your partner see it, write an equation of a line so that both the x-intercept and the y-intercept are each between -3 and 3. Graph your equation on one of the coordinate systems.

Your inequality



2. Still without letting your partner see it, write an inequality for which your equation is the related equation. In other words, your line should be the boundary between solutions and non-solutions. Shade the solutions on your graph.

3. Take turns stating coordinates of points. Your partner will tell you whether your guess is a solution to their inequality. After each partner has stated a point, each may guess what the other's inequality is. If neither guesses correctly, play continues. Use the other coordinate system to keep track of your guesses.



Activity 2: Representation of Inequalities

Your teacher will give you a set of cards. Take turns with your partner to match a group of four cards that contain: a situation, an inequality that represents it, a graph that represents the solution region, and a solution written as a coordinate pair.

For each match that you find, explain to your partner how you know it's a match. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

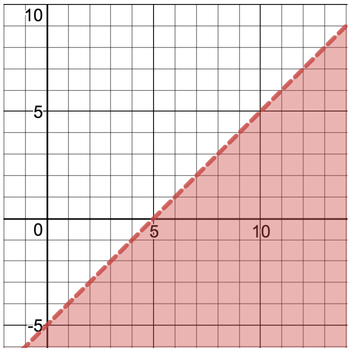
Record your matches.

Group 1 • situation: perimeter of a rectangle • inequality: • a solution: • sketch of graph:	Group 2 • situation: jar of coins • inequality: • a solution: • sketch of graph:
Group 3 • situation: honey and jam • inequality: • a solution: • sketch of graph:	Group 4 • situation: a school trip • inequality: • a solution: • sketch of graph:

Lesson Debrief

Lesson 17 Summary and Glossary

Suppose we want to find the solution to $x - y > 5$.



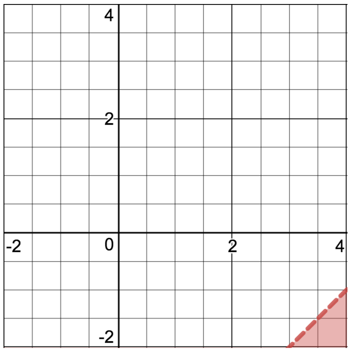
Graphing technology can help us graph the solution to an inequality in two variables.

Many graphing tools allow us to enter inequalities such as $x - y > 5$ and will show the solution region, as shown here.

Some tools, however, may require the inequalities to be in slope-intercept form or another form before displaying the solution region. Be sure to learn how to use the graphing technology available in your classroom.

Although graphing using technology is efficient, we still need to analyze the graph with care. Here are some things to consider:

- The graphing window. If the graphing window is too small, we may not be able to really see the solution region or the boundary line, as shown here.



- The meaning of solution points in the situation. For example, if x and y represent the lengths of two sides of a rectangle, then only positive values of x and y (or points in the first quadrant) make sense in the situation.

Unit 3 Lesson 17 Practice Problems

1. This year, students in the 9th grade are collecting dimes and quarters for a school fundraiser. They are trying to collect more money than the students who were in the 9th grade last year. The students in 9th grade last year collected $143.88.

   Using d to represent the number of dimes collected and q to represent the number of quarters, which statement **best** represents this situation?

   a. $0.25d + 0.1q \geq 143.88$

   b. $0.25q + 0.1d \geq 143.88$

   c. $0.25d + 0.1q > 143.88$

   d. $0.25q + 0.1d > 143.88$

2. A farmer is creating a budget for planting soybeans and wheat. Planting soybeans costs $200 per acre, and planting wheat costs $500 per acre. He wants to spend no more than $100,000 planting soybeans and wheat.

   a. Write an inequality to describe the constraints. Specify what each variable represents.

   b. Name one solution to the inequality and explain what it represents in that situation.

3. Priya is ordering dried chili peppers and corn husks for her cooking class. Chili peppers cost \$16.95 per pound, and corn husks cost \$6.49 per pound.

   Priya spends less than \$50 on d pounds of dried chili peppers and h pounds of corn husks.

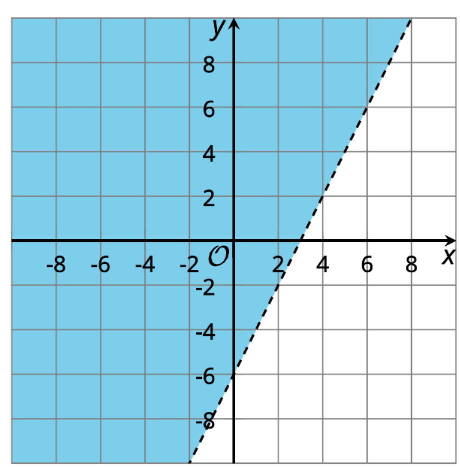
   Here is a graph that represents this situation.

   a. Write an inequality that represents this situation.

   b. Can Priya purchase 2 pounds of dried chili peppers and 4 pounds of corn husks and spend less than \$50? Explain your reasoning.

   c. Can Priya purchase 1.5 pounds of dried chili peppers and 3 pounds of corn husks and spend less than \$50? Explain your reasoning.

4. Which inequality is represented by the graph?



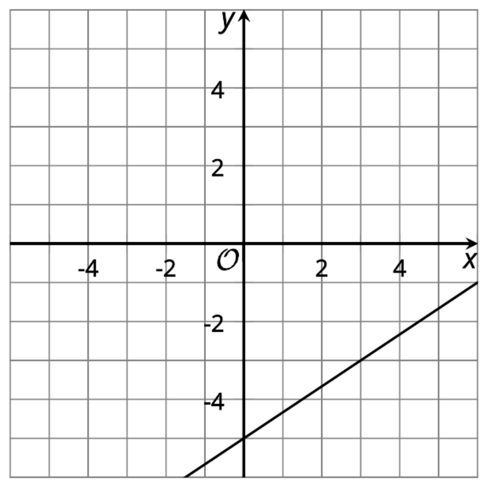
   a. $4x–2y > 12$

   b. $4x–2y < 12$

   c. $4x + 2y > 12$

   d. $4x + 2y < 12$

5. Here is a graph of the equation: $2x - 3y = 15$.



a. Are the points $(1.5, -4)$ and $(4, -4)$ solutions to the equation? Explain or show how you know.

b. Check if each of these points is a solution to the inequality $2x - 3y < 15$:

- $(0, -5)$
- $(4, -2)$
- $(2, -4)$
- $(5, -1)$

c. Shade the solutions to the inequality.

d. Are the points on the line included in the solution region? Explain how you know.

(From Unit 3, Lesson 16)

6. A store sells notepads in packages of 24 and packages of 6. The organizers of a conference need to prepare at least 200 notepads for the event.

   a. Would they have enough notepads if they bought these quantities?

      i. Seven packages of 24 and one package of 6

      ii. Five packages of 24 and fifteen packages of 6

   b. Write an inequality to represent the relationship between the number of large and small packages of notepads and the number of notepads needed for the event.

   c. Use graphing technology to graph the solution set to the inequality. Then, use the graph to name two other possible combinations of large and small packages of notepads that will meet the number of notepads needed for the event.

(From Unit 3, Lesson 15)

7. Triangle ABC has vertices $A\ (6,\ 3)$, $B\ (1,\ 7)$, and $C\ (11,\ -1)$. Is triangle ABC equilateral, isosceles, or scalene? How do you know?

(From Unit 3, Lessons 13 & 14)

8. Elena is solving this system of equations: $\begin{cases} 10x - 6y = 16 \\ 5x - 3y = 8 \end{cases}$

   She multiplies the second equation by 2, then subtracts the resulting equation from the first. To her surprise, she gets the equation $0 = 0$.

   What is special about this system of equations? Why does she get this result and what does it mean about the solutions? (If you are not sure, try graphing them.)

(From Unit 3, Lesson 12)

9. Jada has a sleeping bag that is rated for 30° F. This means that if the temperature outside is at least 30° F, Jada will be able to stay warm in her sleeping bag.

    a. Write an inequality that represents the outdoor temperature at which Jada will be able to stay warm in her sleeping bag.

    b. Write an inequality that represents the outdoor temperature at which a thicker or warmer sleeping bag would be needed to keep Jada warm.

(From Unit 2)

10. What is the solution set to this inequality: $6x - 20 > 3(2 - x) + 6x - 2$?

(From Unit 2)

11. The school band director determined from past experience that if they charge t dollars for a ticket to the concert, they can expect attendance of $1000 - 50t$. The director used this model to figure out that the ticket price needs to be $8 greater in order for at least 600 to attend. Do you agree with this claim? Why or why not?[2]

(Addressing NC.7.EE.4b)

Lesson 18: Solutions to Systems of Linear Inequalities in Two Variables

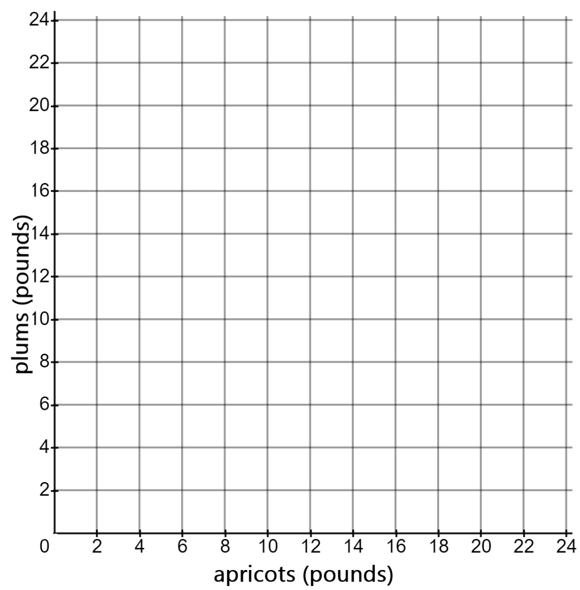
Learning Targets

- When given descriptions and graphs that represent two different constraints, I can find values that satisfy each constraint individually and values that satisfy both constraints at once.
- I know what is meant by "the solutions to a system of inequalities" and can describe the graphs that represent the solutions.
- I can write a system of inequalities to describe a situation, find the solutions by graphing, and interpret points in the solution region.

Bridge

Jada goes to an orchard to pick plums and apricots to make jam. She picks 20 pounds of fruit altogether.

1. Write an equation that represents the situation. Then graph the equation.



2. If Jada picks a pound of apricots, how many pounds of plums does she pick?

3. Does the point $(5, 16)$ represent a combination of pounds of plums and pounds of apricots that satisfies the constraints in this situation? Explain your reasoning.

Warm-up: A Silly Riddle

Here is a riddle: “I am thinking of two numbers that add up to 20. The difference between them is 6. What are the two numbers?”

1. Name any pair of numbers whose sum is 20.

2. Name any pair of numbers whose difference is 6.

3. The riddle can be represented with two equations. Write the equations.

4. Solve the riddle. Explain or show your reasoning.

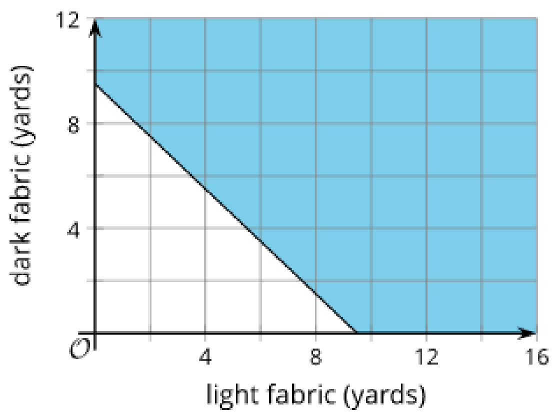
Activity 1: A Quilting Project

To make a quilt, a quilter is buying fabric in two colors, light and dark. He needs at least 9.5 yards of fabric in total.

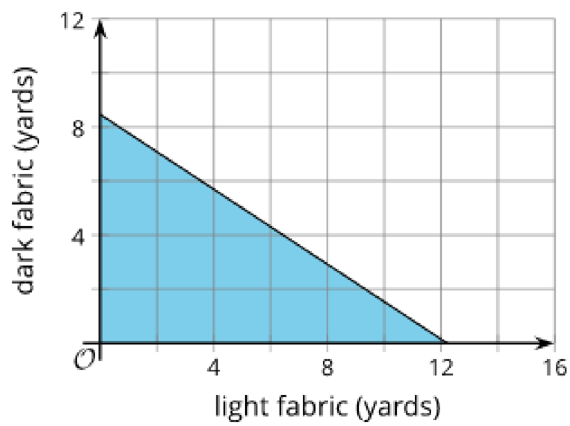
The light color costs \$9 a yard. The dark color costs \$13 a yard. The quilter can spend up to \$110 on fabric.

Here are two graphs that represent the two constraints.

Graph A



Graph B



1. Write an inequality to represent the length constraint. Let x represent the yards of light fabric and y represent the yards of dark fabric.

2. Select **all** the pairs that satisfy the length constraint.

   a. $(5,5)$

   b. $(2.5,4.5)$

   c. $(7.5,3.5)$

   d. $(12,10)$

3. Write an inequality to represent the cost constraint.

4. Select **all** the pairs that satisfy the cost constraint.

   a. $(1,1)$

   b. $(4,5)$

   c. $(8,3)$

   d. $(10,1)$

5. Explain why $(2, 2)$ satisfies the cost constraint, but not the length constraint.

6. Find at least one pair of numbers that satisfies **both** constraints. Be prepared to explain how you know.

7. What does the pair of numbers represent in this situation?

Activity 2: Remember These Situations?

Here are some situations you have seen before. Answer the questions for one situation.

Bank Accounts

- A customer opens a checking account and a savings account at a bank. They will deposit a maximum of $600, some in the checking account and some in the savings account. (They might not deposit all of it and instead keep some of the money as cash.)
- The bank requires a minimum balance of $50 in the savings account. It does not matter how much money is kept in the checking account.

Concert Tickets

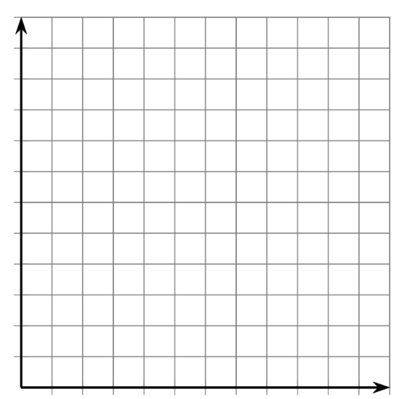
- Two kinds of tickets to an outdoor concert were sold: lawn tickets and seat tickets. Fewer than 400 tickets in total were sold.
- Lawn tickets cost $30 each and seat tickets cost $50 each. The organizers want to make at least $14,000 from ticket sales.

Advertising Packages

- An advertising agency offers two packages for small businesses who need advertising services. A basic package includes only design services. A premium package includes design and promotion. The agency's goal is to sell at least 60 packages in total.
- The basic advertising package has a value of $1,000 and the premium package has a value of $2,500. The goal of the agency is to sell more than $60,000 worth of small-business advertising packages.

1. Write a system of inequalities to represent the constraints. Specify what each variable represents.

2. Use technology to graph the inequalities and sketch the solution regions. Include labels and scales for the axes.



3. Identify a solution to the system. Explain what the numbers mean in the situation.

Activity 3: A Scavenger Hunt

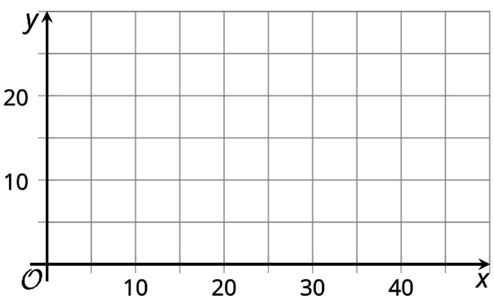
Members of a high school math club are doing a scavenger hunt. Three items are hidden in the park, which is a rectangle that measures 50 meters by 20 meters.

- The clues are written as systems of inequalities. One system has no solutions.
- The locations of the items can be narrowed down by solving the systems. A coordinate plane can be used to describe the solutions.

Can you find the hidden items? Sketch a graph to show where each item could be hidden.

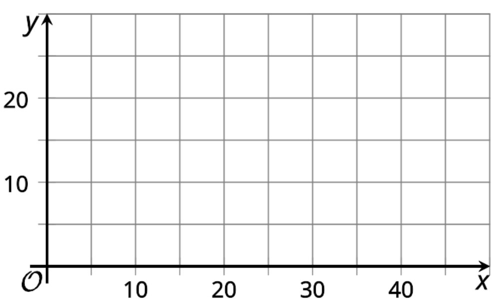
Clue 1:

$$\begin{cases} y > 14 \\ x < 10 \end{cases}$$



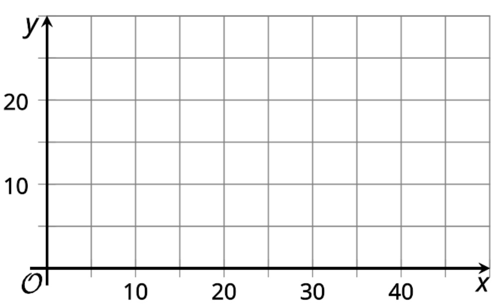
Clue 2:

$$\begin{cases} x + y < 20 \\ x > 6 \end{cases}$$



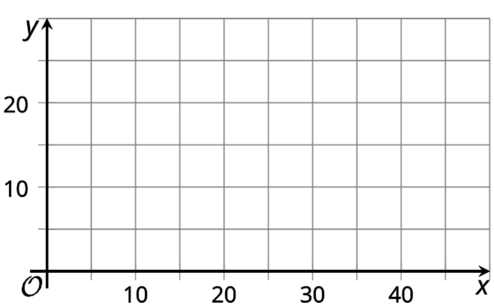
Clue 3:

$$\begin{cases} y < -2x + 20 \\ y < -2x + 10 \end{cases}$$



Clue 4:

$$\begin{cases} y \geq x + 10 \\ x > y \end{cases}$$



Are You Ready For More?

Two non-negative numbers x and y satisfy $x + y \leq 1$.

1. Find a second inequality, also using x and y values greater than or equal to zero, to make a system of inequalities with exactly one solution.

2. Find as many ways to answer this question as you can.

Lesson Debrief

Lesson 18 Summary and Glossary

In this lesson, we used two linear inequalities in two variables to represent the constraints in a situation. Each pair of inequalities forms a **system of inequalities**.

System of inequalities: Two or more inequalities that represent the constraints in the same situation.

A **solution to the system** is any (x, y) pair that makes both inequalities true, or any pair of values that simultaneously meet both constraints in the situation. The set of all solutions to the system is often best represented by a region on a graph.

Solutions to a system of inequalities: All pairs of values that make the inequalities in a system true. The solutions to a system of inequalities can be represented by the points in the region where the graphs of the two inequalities overlap. We also call these solutions the **solution set**.

Suppose there are two numbers, x and y, and there are two things we know about them:

- The value of one number is more than double the value of the other.
- The sum of the two numbers is less than 10.

We can represent these constraints with a system of inequalities. $\begin{cases} y > 2x \\ x + y < 10 \end{cases}$

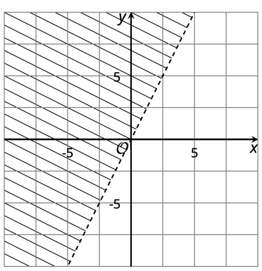
There are many possible pairs of numbers that meet the first constraint, for example: 1 and 3, or 4 and 9.

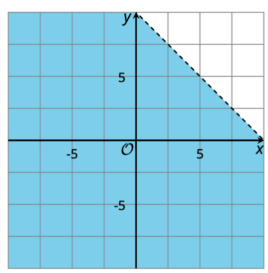
The same can be said about the second constraint, for example: 1 and 3, or 2.4 and 7.5.

The pair $x = 1$ and $y = 3$ meets both constraints, so it is a solution to the system.

The pair $x = 4$ and $y = 9$ meets the first constraint but not the second ($9 > 2(4)$ is a true statement, but $4 + 9 < 10$ is not true.)

Remember that graphing is a great way to show all the possible solutions to an inequality, so let's graph the solution region for each inequality.

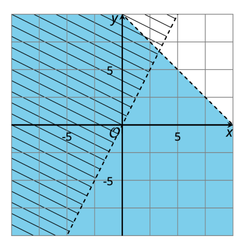




Because we are looking for a pair of numbers that meet both constraints or make both inequalities true at the same time, we want to find points that are in the solution regions of both graphs.

To do that, we can graph both inequalities on the same coordinate plane.

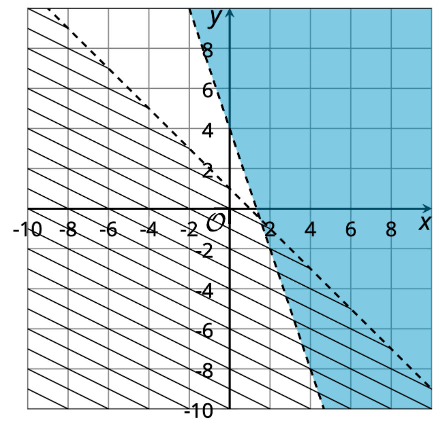
The solution set to the system of inequalities is represented by the region where the two graphs overlap.



Unit 3 Lesson 18 Practice Problems

1. Two inequalities are graphed on the same coordinate plane.

   Which region represents the solution to the system of the two inequalities?



2. Select **all** the pairs of x and y that are solutions to the system of inequalities: $\begin{cases} y \leq -2x+6 \\ x - y < 6 \end{cases}$

   a. $x=0, y=0$

   b. $x=-5, y=-15$

   c. $x=4, y=-2$

   d. $x=3, y=0$

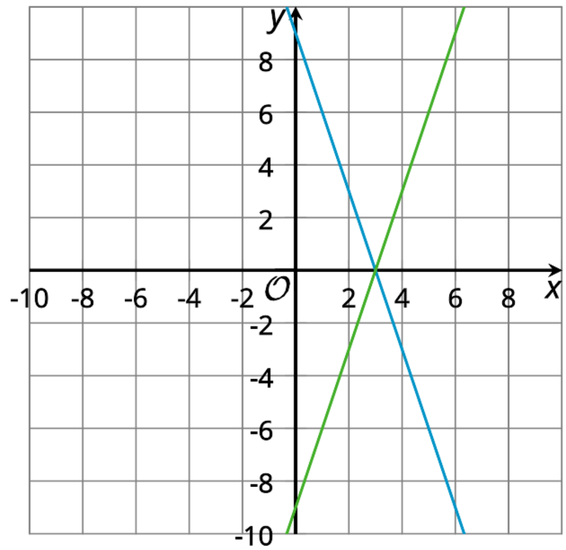
   e. $x=10, y=0$

3. Jada has $200 to spend on flowers for a school celebration. She decides that the only flowers that she wants to buy are roses and carnations. Roses cost $1.45 each and carnations cost $0.65 each. Jada buys enough roses so that each of the 75 people attending the event can take home at least one rose.

   a. Write an inequality to represent the constraint that every person takes home at least one rose.

   b. Write an inequality to represent the cost constraint.

4. Here are the graphs of the equations $3x + y = 9$ and $3x - y = 9$ on the same coordinate plane.



a. Label each graph with the equation it represents.

b. Identify the region that represents the solution set to $3x + y < 9$. Is the boundary line a part of the solution? Use a colored pencil or cross-hatching to shade the region.

c. Identify the region that represents the solution set to $3x - y < 9$. Is the boundary line a part of the solution? Use a different colored pencil or cross-hatching to shade the region.

d. Identify a point that is a solution to both $3x + y < 9$ and $3x - y < 9$.

5. In physical education class, Mai takes 10 free throws and 10 jump shots. She earns 1 point for each free throw she makes and 2 points for each jump shot she makes. The greatest number of points that she can earn is 30.

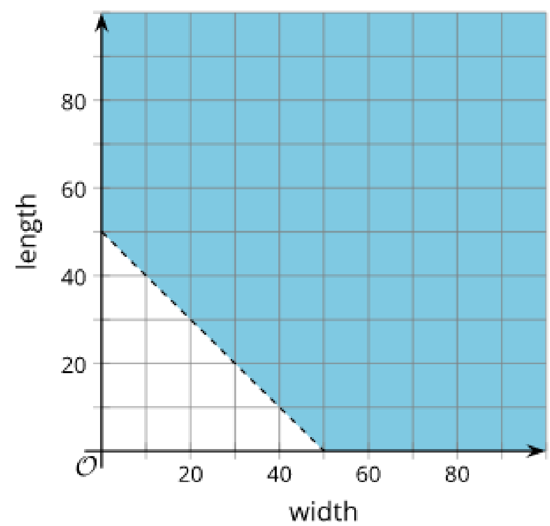
    a. Write an inequality to describe the constraints. Specify what each variable represents.

    b. Name one solution to the inequality and explain what it represents in that situation.

(From Unit 3, Lesson 17)

6. A rectangle with a width of w and a length of l has a perimeter greater than 100.

    Here is a graph that represents this situation.



    a. Write an inequality that represents this situation.

    b. Can the rectangle have a width of 45 and a length of 10? Explain your reasoning.

    c. Can the rectangle have a width of 30 and a length of 20? Explain your reasoning.

(From Unit 3, Lesson 17)

7. Which coordinate pair is a solution to the inequality $4x - 2y < 22$?

    a. $(4, -3)$

    b. $(4, 3)$

    c. $(8, -3)$

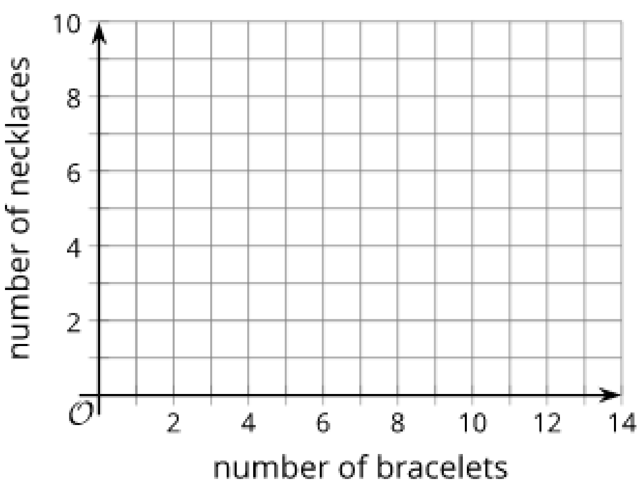
    d. $(8, 3)$

(From Unit 3, Lesson 16)

8. Elena is considering buying bracelets and necklaces as gifts for her friends. Bracelets cost \$3, and necklaces cost \$5. She can spend no more than \$30 on the gifts.

    a. Write an inequality to represent the number of bracelets, b, and the number of necklaces n, she could buy while sticking to her budget.

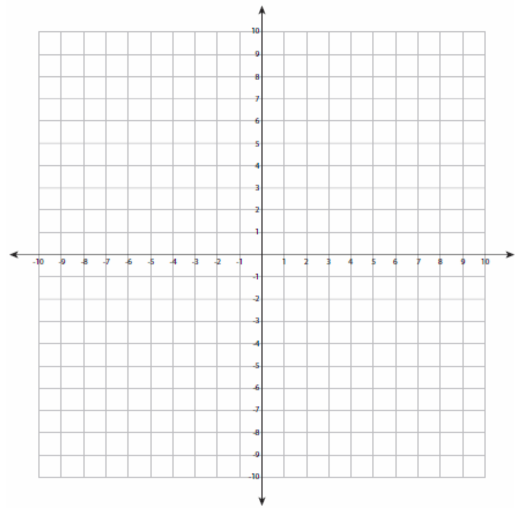
    b. Graph the solutions to the inequality on the coordinate plane.



    c. Explain how we could check if the boundary is included or excluded from the solution set.

(From Unit 3, Lesson 15)

9. Point R is at $(-3,\ 6)$. Find the distance from point R to:



   a. The origin

   b. The x-axis

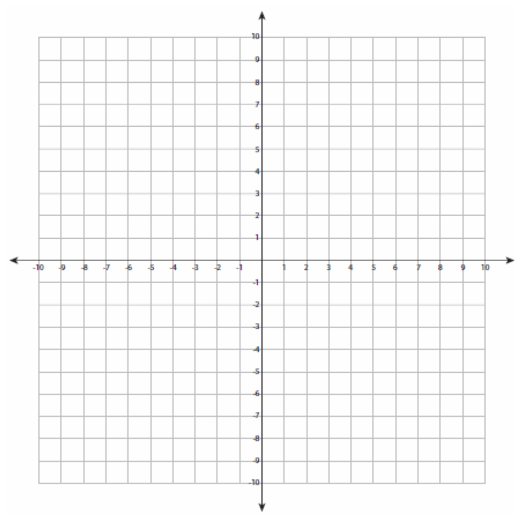
   c. The y-axis

   d. Point $(1,\ -2)$

   e. Which distance was the farthest? How could you have predicted that using the coordinates?

(From Unit 3, Lessons 13 & 14)

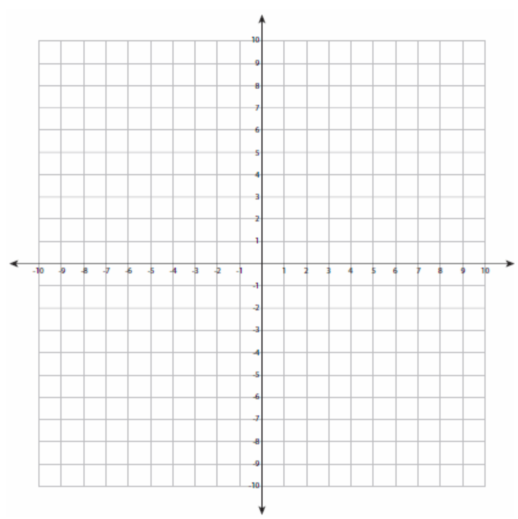
10. The perpendicular bisector of a line segment is a line perpendicular to the segment that passes through its midpoint. What is an equation for the perpendicular bisector of segment XY, with point $X\ (2,\ -6)$ and $Y\ (5,\ -5)$?



(From Unit 3, Lessons 6 and 13 & 14)

11. A triangle has vertices $D\,(1,5)$, $O\,(7,9)$, and $G\,(7,2)$.

    Is DOG equilateral, isosceles, or neither? How do you know?



(From Unit 3, Lesson 7)

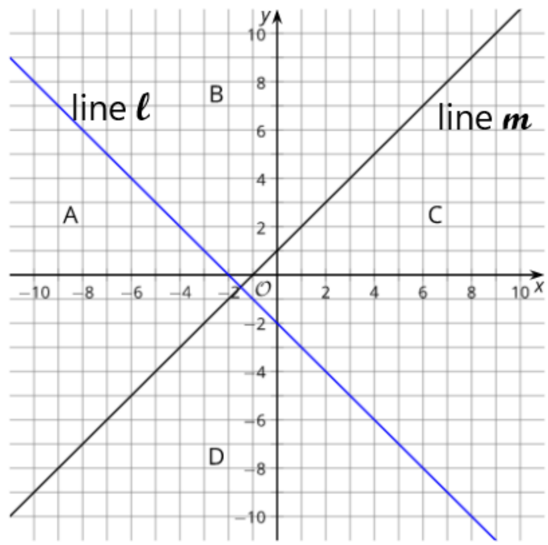
Lesson 19: Solving Problems with Systems of Linear Inequalities in Two Variables

Learning Target

- I can explain how to tell if a point on the boundary of the graph of the solutions to a system of inequalities is a solution or not.

Bridge

1. The graph shows the lines $y = x + 1$ and $y = -x - 2$. Which line represents $y = x + 1$?



2. Write a coordinate pair for a point in region A.

3. For which of the following systems is the point a solution? Explain how you know.

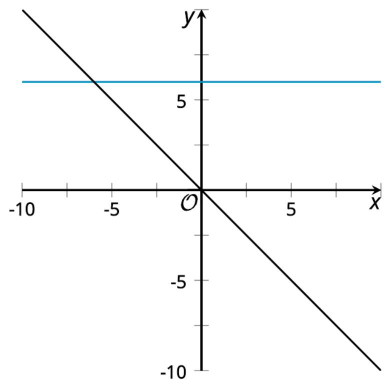
a. $\begin{cases} y \geq x + 1 \\ y \leq -x - 2 \end{cases}$

b. $\begin{cases} y \geq x + 1 \\ y \geq -x - 2 \end{cases}$

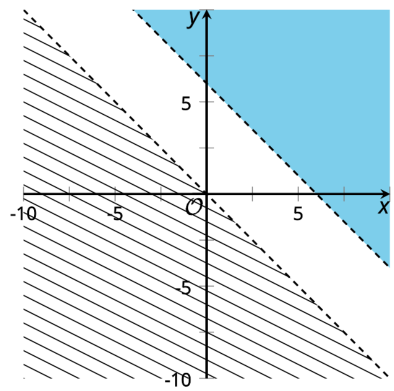
Warm-up: Graphs of Solutions

Which one doesn't belong? Explain your reasoning.

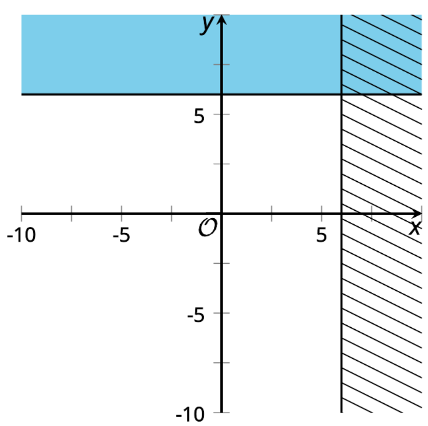
a.



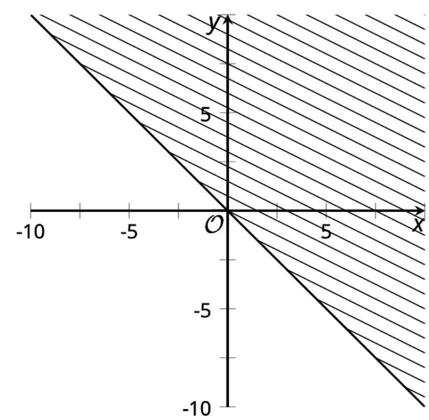
b.



c.

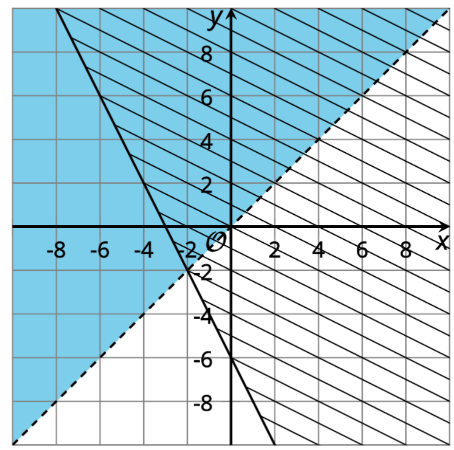


d.



Activity 1: Focusing on the Details

Here are the graphs of the inequalities in this system: $\begin{cases} x < y \\ y \geq -2x - 6 \end{cases}$

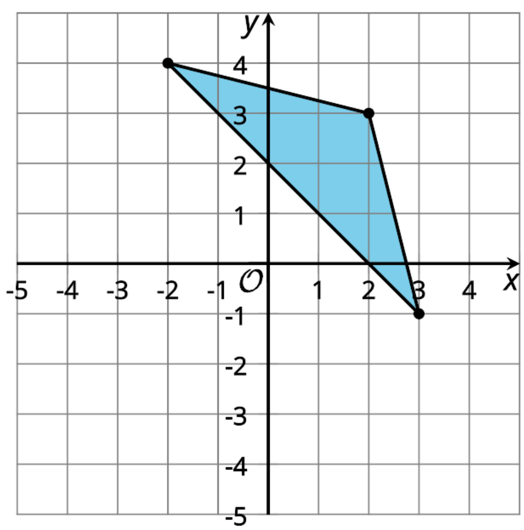


Decide whether each point is a solution to the system. Be prepared to explain how you know.

1. $(3, -5)$
2. $(0, 5)$
3. $(-6, 6)$
4. $(3, 3)$
5. $(-2, -2)$

Are You Ready For More?

Find a system of inequalities with this triangle as its set of solutions.

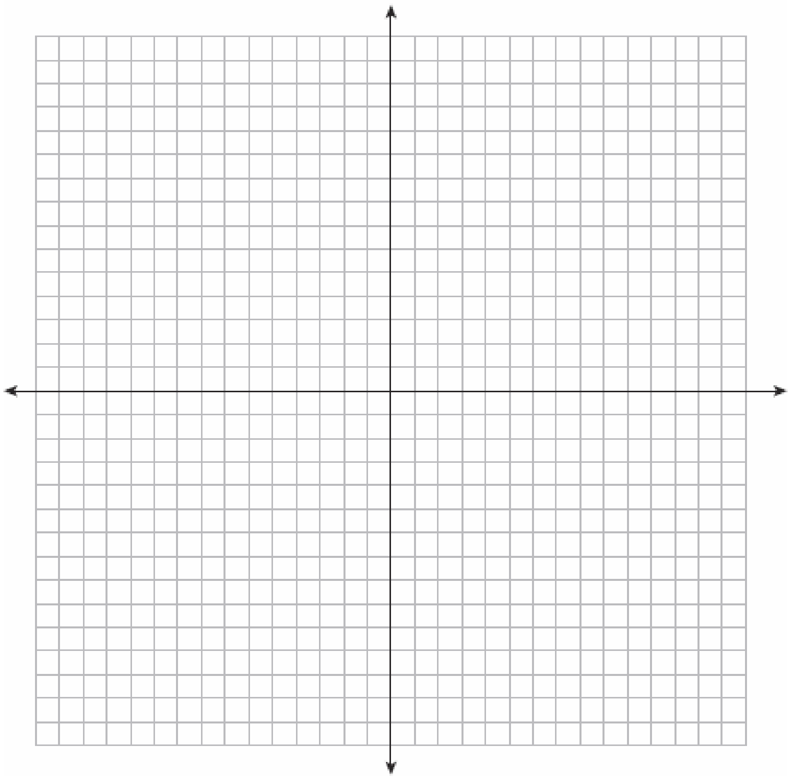


Activity 2: Pet Sitters[1]

Andre and Elena are starting a pet sitting business to care for cats and dogs while their owners are on vacation.

- Space: Cat pens will require 6 square feet of space, while dog runs require 24 square feet. Andre and Elena have up to 360 square feet available in the storage shed for pens and runs, while still leaving enough room to move around the cages.
- Startup Costs: Andre and Elena plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

1. Create a system of inequalities to represent the constraints of space and startup costs.



2. Graph the system of inequalities.

3. Identify four different points and interpret what each point means in terms of the situation. Choose one point of each of these types:
   a. within the intersecting regions
   b. on one of the boundary lines
   c. on the other boundary line
   d. the intersection of the two lines

[1] Adapted from Secondary One Mathematics Vision project http://www.mathematicsvisionproject.org, licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0)

Are You Ready For More?

Andre and Elena want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

What combination of the number of cats and number of dogs would maximize the income? Show or explain your reasoning.

Lesson Debrief

Lesson 19 Summary and Glossary

A family has at most $25 to spend on activities at Fun Zone. It costs $10 an hour to use the trampolines and $5 an hour to use the pool. The family can stay less than 4 hours.

What are some combinations of trampoline time and pool time that the family could choose given their constraints?

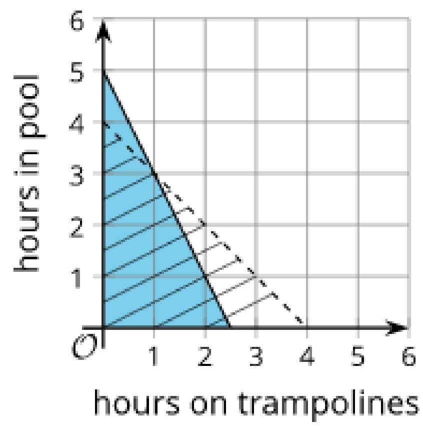
We could find some combinations by trial and error, but writing a system of inequalities and graphing the solution would allow us to see all the possible combinations.

Let t represent the time, in hours, on the trampolines and p represent the time, in hours, in the pool.

The constraints can be represented with the system of inequalities:

$$\begin{cases} 10t + 5p \leq 25 \\ t + p < 4 \end{cases}$$

Here are graphs of the inequalities in the system.



The solution set to the system is represented by the region where the shaded parts of the two graphs overlap. Any point in that region is a pair of times that meet both the time and budget constraints.

The graphs give us a complete picture of the possible solutions.

- Can the family spend 1 hour on the trampolines and 3 hours in the pool?

  No. We can reason that this is too much time because those times add up to 4 hours, and the family wants to spend *less than* 4 hours. But we can also see that the point $(1, 3)$ lies on the dashed line of one graph, so it is not a solution.

- Can the family spend 2 hours on the trampolines and 1.5 hours in the pool?

  No. We know that these two times add up to less than 4 hours, but to find out the cost, we need to calculate $10(2) + 5(1.5)$, which is 27.5 and is more than the budget.

  It may be easier to see that this combination is not an option by noticing that the point $(2, 1.5)$ is in the region with line shading, but not in the region with solid shading. This means it meets one constraint but not the other.

Unit 3 Lesson 19 Practice Problems

1. Jada has p pennies and n nickels that add up to more than 40 cents. She has fewer than 20 coins altogether.

    a. Write a system of inequalities that represents how many pennies and nickels that Jada could have.

    b. Is it possible that Jada has each of the following combinations of coins? If so, explain or show how you know. If not, state which constraint(s)—the amount of money or the number of coins—it does not meet.

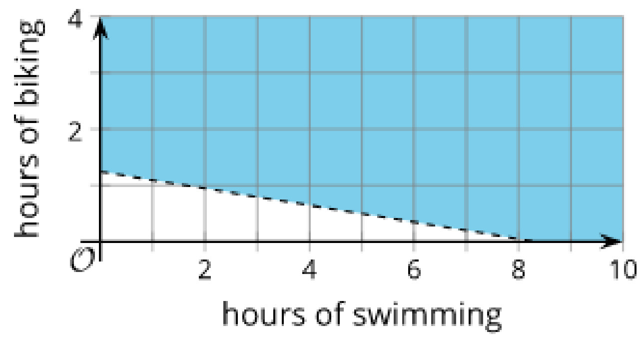
    - 15 pennies and 5 nickels

    - 16 pennies and 2 nickels

    - 10 pennies and 8 nickels

2. A triathlon athlete swims at an average rate 2.4 miles per hour, and bikes at an average rate of 16.1 miles per hour. At the end of one training session, she swam and biked more than 20 miles in total.

   The inequality $2.4s + 16.1b > 20$ and this graph represent the relationship between the hours of swimming, s, the hours of biking, h, and the total distance the athlete could have traveled in miles.



   Mai said, "I'm not sure the graph is right. For example, the point $(10, 3)$ is in the shaded region, but it's not realistic for an athlete to swim for 10 hours and bike for 3 hours in a training session! I think triathlon athletes generally train for no more than 2 hours a day."

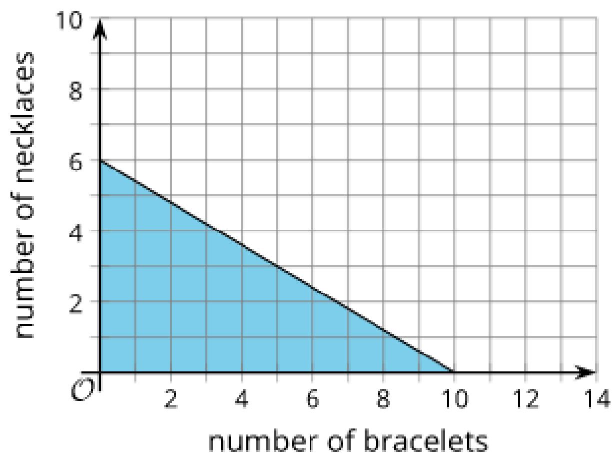
   a. Write an inequality to represent Mai's last statement.

   b. Graph the solution set to your inequality.

   c. Determine a possible combination of swimming and biking times that meet both the distance and the time constraints in this situation.

3. Elena is considering buying bracelets and necklaces as gifts for her friends. Bracelets cost \$3, and necklaces cost \$5. She can spend no more than \$30 on the gifts. Elena needs at least 7 gift items.

   This graph represents the inequality $3b + 5n \leq 30$, which describes the cost constraint in this situation.
   Let b represent the number of bracelets and n the number of necklaces.



   a. Write an inequality that represents the number of gift items that Elena needs.

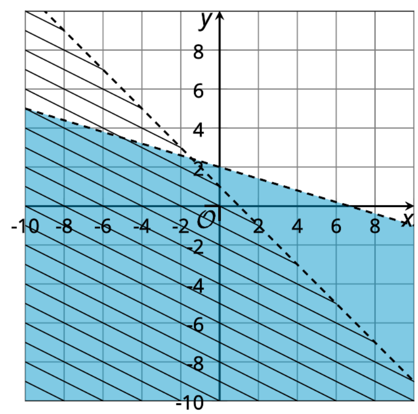
   b. On the same coordinate plane, graph the solution set to the inequality you wrote.

   c. Use the graphs to find at least two possible combinations of bracelets and necklaces Elena could buy.

   d. Explain how the graphs show that the combination of 2 bracelets and 5 necklaces meet one constraint in the situation but not the other constraint.

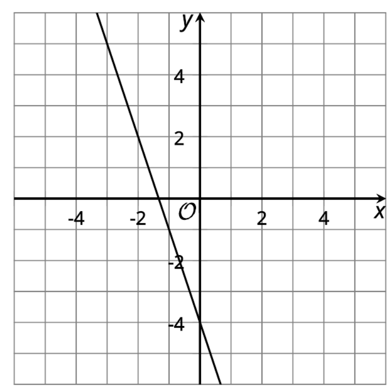
4. Two inequalities are graphed on the same coordinate plane.

   Which region represents the solution to the system of the two inequalities?



(From Unit 3, Lesson 18)

5. Here is a graph of the equation $6x + 2y = -8$.



   a. Are the points $(1.5, -4)$ and $(0, -4)$ solutions to the equation? Explain or show how you know.

   b. Check if each of these points is a solution to the inequality $6x + 2y \leq -8$:

   $(-2, 2)$ $(4, -2)$ $(0, 0)$ $(-4, -4)$

   c. Shade the solutions to the inequality.

   d. Are the points on the line included in the solution region? Explain how you know.

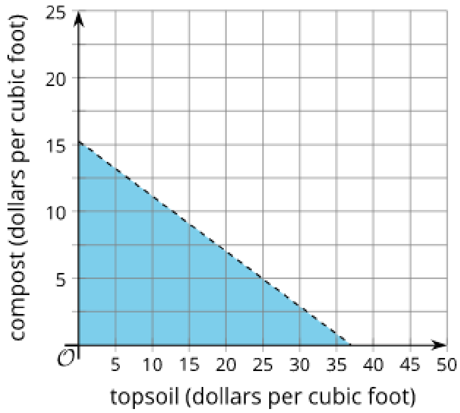
(From Unit 3, Lesson 16)

6. A gardener is buying some topsoil and compost to fill his garden. His budget is \$70. Topsoil costs \$1.89 per cubic foot, and compost costs \$4.59 per cubic foot.

   Select **all** statements or representations that correctly describe the gardener's constraints in this situation. Let t represent the cubic feet of topsoil and c the cubic feet of compost.

   a. The combination of 7.5 cubic feet of topsoil and 12 cubic feet of compost is within the gardener's budget.

   b. If the line represents the equation $1.89t + 4.59c = 70$, this graph represents the solutions to the gardener's budget constraint.

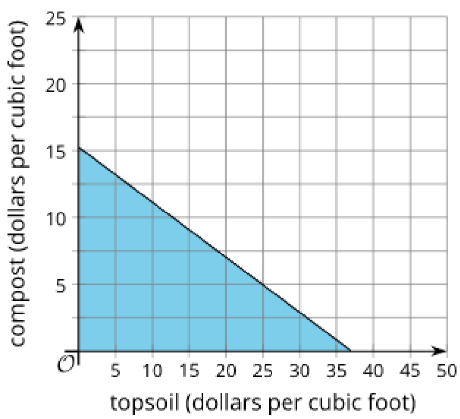


   c. $1.89t + 4.59c \geq 70$

   d. The combination of 5 cubic feet of topsoil and 20 cubic feet of compost is within the gardener's budget.

   e. $1.89t + 4.59c \leq 70$

   f. If the line represents the equation $1.89t + 4.59c = 70$, this graph represents the solutions to the gardener's budget constraint.



(From Unit 3, Lesson 15)

7. Fill in the missing coordinates: S is the midpoint of segment UA, with points $U\ (6, y)$, $S\ (x, -4)$, and $A\ (-1, -1)$.

(From Unit 3, Lessons 13 & 14)

8. Priya writes the equation $y = -\frac{1}{2}x - 7$. Write an equation that has:

   a. exactly one solution in common with Priya's equation

   b. no solutions in common with Priya's equation

   c. infinitely many solutions in common with Priya's equation, but looks different than hers

(From Unit 3, Lesson 12)

9. Han is planning a trip to South Africa, and he wants to learn about rugby. It's one of the most popular sports in South Africa!

   In the first game Han watches, the South African National Team, the Springboks, scored 59 total points, with nine tries and seven conversions. Their opponent, the Wallabies from Australia, scored 49 total points, with nine tries and two conversions.

   If each try, *t*, and each conversion, *c*, is worth the same amount of points, how many points is one try and one conversion worth together?

(From Unit 3, Lesson 10)

10. One side of a square has endpoints $A\,(1,4)$ and $B\,(4,0)$. What are the equations of the lines containing the two adjacent sides to side AB?

(From Unit 3, Lesson 6)

11. Lin is taking a trip to Europe, and she needs to convert her American dollars to European currency. When she's in England, she'll need to use the currency, pounds, and when she's in France, she'll need to use the currency, euros. The current exchange rate is 0.71 pounds per dollar and 0.82 euros per dollar. Lin wants to bring $1,000 to spend.

    a. Write an equation describing the number of pounds, p, and euros, e, Lin can exchange for $1,000.

    b. If Lin exchanges the money in all pounds, how many will she get? Explain below how this could be represented on a graph.

    c. If Lin exchanges the money in all euros, how many will she get? Explain below how this could be represented on a graph.

(From Unit 3, Lesson 2)

Lesson 20: Modeling with Systems of Inequalities in Two Variables

Learning Targets

- I can interpret inequalities and graphs in a mathematical model.
- I know how to choose variables, specify the constraints, and write inequalities to create a mathematical model for a given situation.

Warm-up: A Solution to Which Inequalities

Is the ordered pair $(5.43, 0)$ a solution to all, some, or none of these inequalities? Be prepared to explain your reasoning.

$x > 0$ $y > 0$ $x \geq 0$ $y \geq 0$

Activity 1: Custom Trail Mix

Here is the nutrition information for some trail mix ingredients:

	calories per gram (kcal)	protein per gram (g)	sugar per gram (g)	fat per gram (g)	fiber per gram (g)
peanuts	5.36	0.21	0.04	0.46	0.07
almonds	5.71	0.18	0.21	0.46	0.07
raisins	3.00	0.03	0.60	0.00	0.05
chocolate pieces	4.76	0.05	0.67	0.19	0.02
shredded coconut	6.67	0.07	0.07	0.67	0.13
sunflower seeds	5.50	0.20	0.03	0.47	0.10
dried cherries	3.25	0.03	0.68	0.00	0.03
walnuts	6.43	0.14	0.04	0.61	0.07

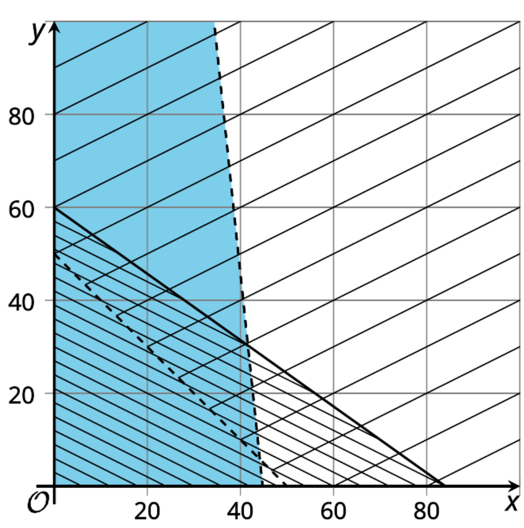
Tyler and Jada each designed their own custom trail mix using two of these ingredients. They wrote inequalities and created graphs to represent their constraints.

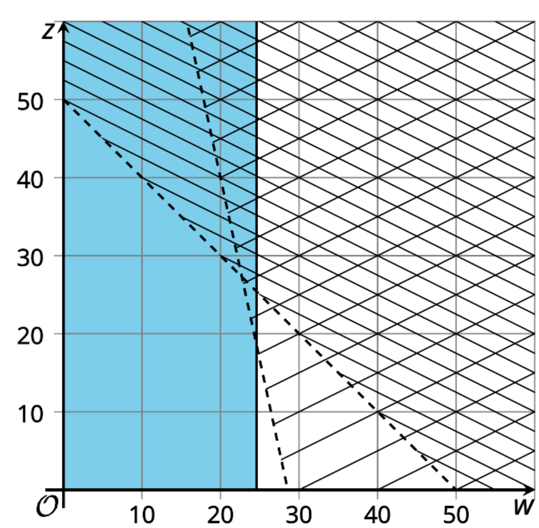
Tyler

- $x + y > 50$
- $4.76x + 6.67y \le 400$
- $0.67x + 0.07y < 30$
- $x > 0$
- $y > 0$

Jada

- $w + z > 50$
- $0.14w + 0.03z > 4$
- $0.61w + 0z \le 15$
- $w > 0$
- $z > 0$





Use the inequalities and graphs to answer these questions about each student's trail mix. Be prepared to explain your reasoning.

1. Which two ingredients did they choose?

2. What do their variables represent?

3. What does each constraint mean?

4. Which graph represents which constraint?

5. Name one possible combination of ingredients for their trail mix.

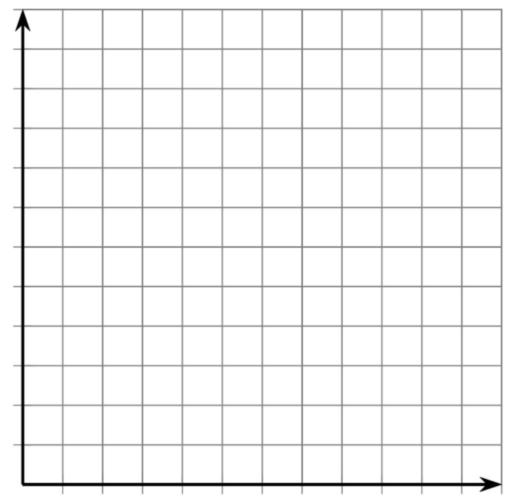
Activity 2: Design Your Own Trail Mix

It's time to design your own trail mix!

1. Choose two ingredients that you like to eat. (You can choose from the ingredients in the previous activity, or you can look up nutrition information for other ingredients.)

2. Think about the constraints for your trail mix. What do you want to be true about its calories, protein, sugar, fat, or fiber?

3. Write inequalities to represent your constraints. Then, graph the inequalities.



4. Is it possible to make a trail mix that meets all your constraints using your ingredients? If not, make changes to your constraints or your ingredients and record them here.

5. Write a possible combination of ingredients for your trail mix.

Lesson Debrief

Lesson 20 Summary and Glossary

Each day a small bakery bakes two types of bread, A and B.

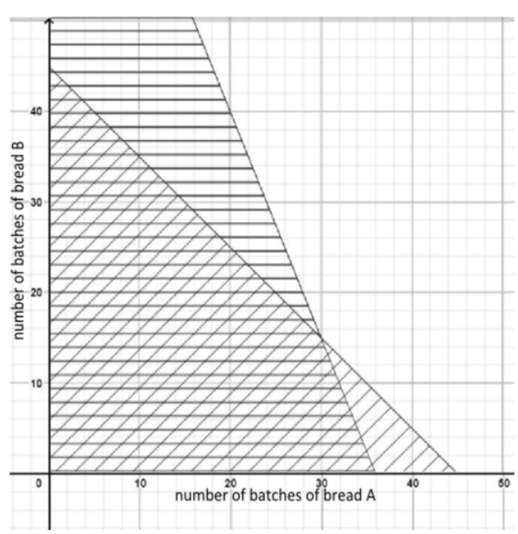
- One batch of bread A uses 5 pounds of oats and 3 pounds of flour.
- One batch of bread B uses 2 pounds of oats and 3 pounds of flour.
- The company has 180 pounds of oats and 135 pounds of flour available each day.

What are the constraints in this situation?

- There is a limit on the amount of oats (180 pounds).
- There is a limit on the amount of flour (135 pounds).
- The number of batches of bread A and bread B must be positive.

Let a represent the number of batches of bread A and let b represent the number of batches of bread B. The constraints can be represented with a system of inequalities.

- $5a + 2b \leq 180$
- $3a + 3b \leq 135$
- $a > 0$
- $b > 0$



Here are the graphs of the inequalities in the system. Notice how the solution set is restricted to the first quadrant.

The constraints of $a > 0$ and $b > 0$ are necessary in this situation for two reasons. First, the number of batches of bread must be a positive number thus greater than 0. Secondly, the bakery will bake both types of bread each day so neither could be equal to 0.

Unit 3 Lesson 20 Practice Problems

1. Four families are visiting Carowinds. For each family, let x be the amount of time spent riding rides and let y be the time spent in Carolina Harbor Waterpark. List one or more ordered pairs that would fit the constraints given by each family.

   a. Clare's family wants to spend at least 4 hours riding the rides, and they want to spend more time in the Carolina Harbor Waterpark than riding rides.

   b. Jada's family wants to be at Carowinds from 4 p.m. to 8 p.m., and they want to spend most of their time riding rides.

   c. Priya's family wants to spend 2 hours at Carolina Harbor Waterpark and 2 hours riding rides.

   d. Diego's family wants to spend no more than 6 hours at Carowinds, and they want to spend at least twice as long riding rides as they spend at Carolina Harbor Waterpark.

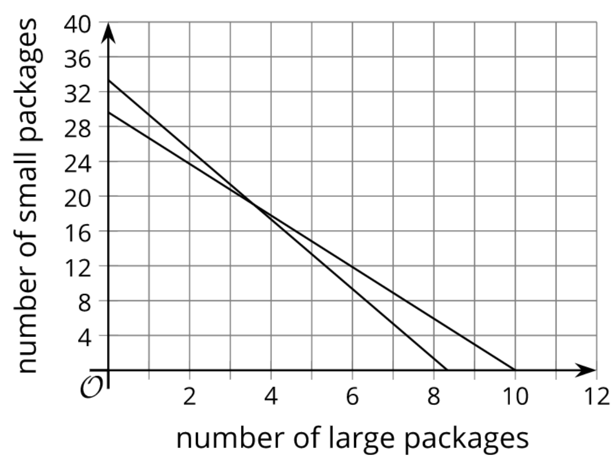
2. The organizers of a conference need to prepare at least 200 notepads for the event and have a budget of $160 for the notepads. A store sells notepads in packages of 24 and packages of 6.

This system of inequalities represent these constraints: $\begin{cases} 24x + 6y \geq 200 \\ 16x + 5.40y \leq 160 \end{cases}$

a. Explain what the second inequality in the system tells us about the situation.

b. Here are incomplete graphs of the inequalities in the system, showing only the boundary lines of the solution regions.

Which graph represents the boundary line of the second inequality?



c. Complete the graphs to show the solution set to the system of inequalities.

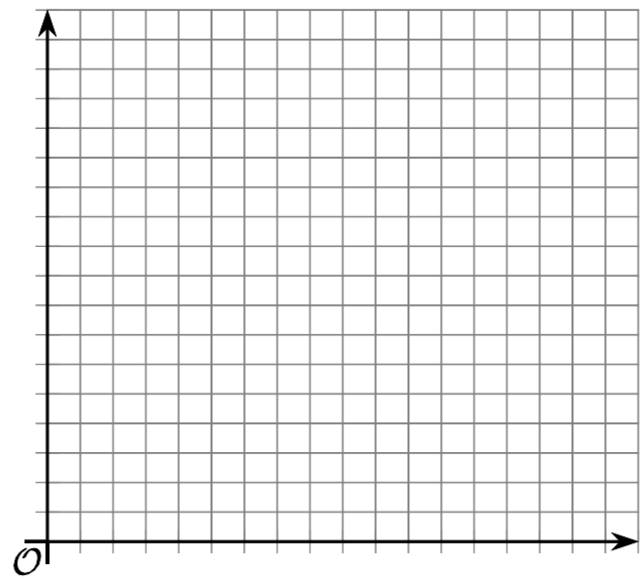
d. Find a possible combination of large and small packages of notepads the organizer could order.

(From Unit 3, Lesson 19)

3. A certain stylist charges $15 for a haircut and $30 for hair coloring. A haircut takes on average 30 minutes, while coloring takes 2 hours. The stylist works up to 8 hours in a day, and she needs to make a minimum of $150 a day to pay for her expenses.

    a. Create a system of inequalities that describes the constraints in this situation. Be sure to specify what each variable represents.

    b. Graph the inequalities and show the solution set.



    c. Identify a point that represents a combination of haircuts and hair-coloring jobs that meets the stylist's requirements.

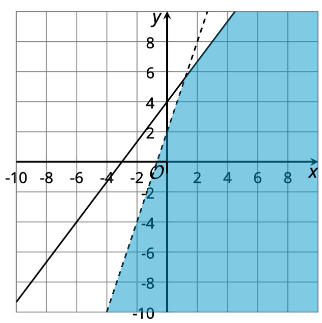
    d. Identify a point that is a solution to the system of inequalities but is not possible or not likely in the situation. Explain why this solution is impossible or unlikely.

(From Unit 3, Lesson 19)

4. Choose the graph that shows the solution to this system:

$$\begin{cases} y > 3x + 2 \\ -4x + 3y \leq 12 \end{cases}$$

a.



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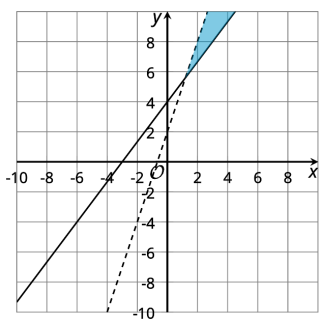

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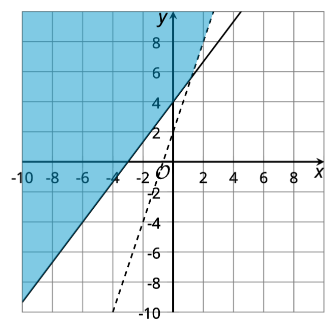

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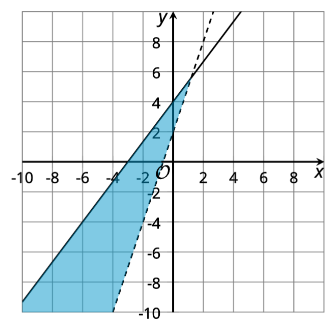
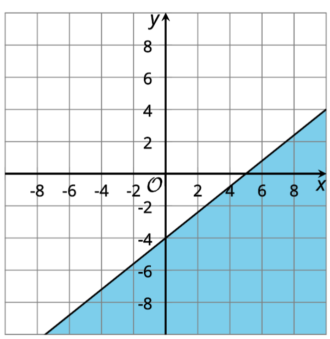
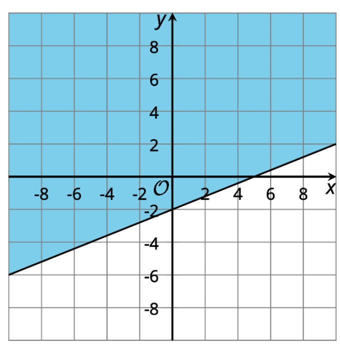
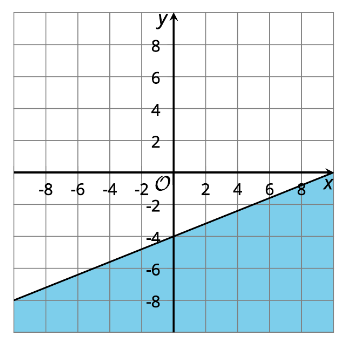
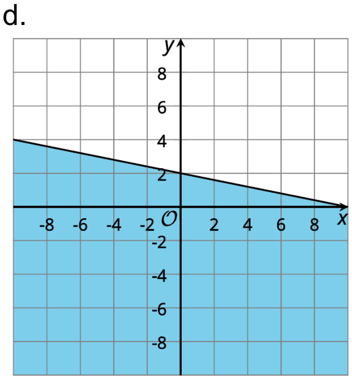
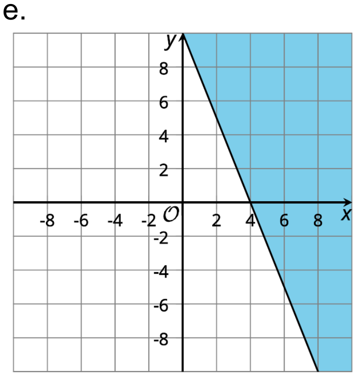


(From Unit 3, Lesson 18)

5. Match each inequality to the graph of its solution.

Inequality	Graph
1. $2x - 5y \geq 20$ 2. $5x + 2y \geq 20$ 3. $4x - 10y \leq 20$ 4. $4x - 5y \geq 20$ 5. $2x + 10y \leq 20$	a.  b.  c.  d.  e. 

(From Unit 3, Lesson 17)

6.

a. In a system of linear equations, the equation for one line is $y = -4.7x + 6.2$. Write a possible equation for the other line if the system has no solutions.

b. In a system of linear equations, the equation for one line is $y + \frac{7}{2} = 2(x - \frac{5}{2})$. Write a possible equation for the other line if the system has one solution.

c. In a system of linear equations, the equation for one line is $5x + 6y = 7$. Write a possible equation for the other line if the system has infinite solutions.

(From Unit 3, Lesson 12)

7. A quadrilateral has vertices $A = (0, 0), B = (4, 6), C = (0, 12),$ and $D = (-4, 6)$. Mai thinks the quadrilateral is a rhombus and Elena thinks the quadrilateral is a square. Do you agree with either of them? Show or explain your reasoning.

(From Unit 3, Lesson 8)

8. Diego and Clare are both saving up for prom - between the ticket, clothes, and dinner, it can get expensive! On March 1, Diego has $30 and can make $14.75 an hour at his part time job, and Clare has $25 and can make $14.75 an hour at her part time job.

    a. Write an equation to represent how much money, m, Diego and Clare will each have after working h hours at their jobs.

    b. Clare thinks they'll have the same amount of money after each of them works 35 hours. Do you agree? Why or why not?

    c. How can a graph of the equations prove whether Clare is correct or not?

(From Unit 3, Lesson 5)

9. Lin's father sends her to the store with $40 to buy balloons and cupcakes for her sister's birthday party. Balloons cost $1.50 each, and cupcakes are sold in packs of 6 for $3.50.

    a. Write an equation to represent how many balloons, b, and packs of cupcakes, c, Lin can buy with $40.

    b. Lin returns home with 30 cupcakes and 15 balloons. Should Lin have change for her father? How do you know?

(From Unit 3, Lesson 1)

Lesson 21: Post-Test Activities

Learning Targets

- I understand the reasoning for and will strive to meet the expectations communicated by my teacher.
- I know my classmates and can recognize the value I will add to this classroom community.

Activity 2: Culture and Mathematics

Take a moment to think about a few of the ways in which your friends, family, and/or community interact with each other. Create a list or share with a partner.

Many cultures are strengthened through interactions revolving around food, music, art, gaming/play, dancing, sports. and public policy.

During this activity you will complete three sections:

1. Discover
    a. Choose one of the following subjects to explore:
        - food
        - music
        - art
        - gaming/play
        - dancing
        - sports
        - public policy

    b. Brainstorm the ways in which your family, friends, and/or community engage in the subject you chose.

    c. Read the related article to understand the connections between your cultural interactions and mathematics. Take notes below.

2. Share
   a. Pair up with one or two classmates who chose the same subject area.
   b. Discuss:
      - One way you engage with this activity that you'd like to share.
      - One thing you learned from the article.
      - One thing you are curious to learn more about around this activity and its connection to math.
   c. Create one chart that represents your group's discussion and display it in the classroom.

3. Learn and Affirm
   a. Walk around the classroom to discover the ways your classmates engage in their friends, families, and/or communities.
   b. Use markers or post-it notes to leave positive notes to your classmates about what they shared.
   c. Reflect: share with a partner or write in your workbook about one thing you found interesting from what your classmates shared.